Strategic Resource Dependence with Learning-by-Doing in the Substitute *

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Abstract

Gerlagh and Liski (2011) show in a model of strategic resource dependence (where a seller of oil faces demand from a buyer who can invest in the production of a substitute) that the standard predictions of the Hotelling nonrenewable resource model are reversed: stocks decline over time, but oil supply increases until the buyer switches to the alternative. Given the considerable evidence that renewable energy (solar energy, wind power, biofuels, etc.) exhibits significant learning-by-doing effects, we extend their model to consider learning in the substitute. We show that, under learning-by-doing, the Hotelling results are restored. Oil prices increase and supply declines over time. In fact, the buyer voluntarily curbs his oil consumption to extend the life of the depleting resource stock owned by the seller. We find that it may be socially efficient to discard part of the oil stock, even though oil is cheaper than the substitute.

Keywords: learning-by-doing, bilateral monopoly, Markov-perfect equilibrium, oil reserves, renewable energy

JEL Codes: C73, Q30, O30

1 Introduction

A major area of research in the economics of nonrenewable resources such as crude oil is the strategic relationship between a buyer and a seller. A buyer may consider developing substitutes that reduce his dependence on the seller. For example, a key reason given for developing domestic shale gas resources in the United States is reduced dependence on foreign oil.[1] However, sellers of crude oil can take such behavior on the part of buyers into account when they price resources. This strategic relationship between a buyer and a seller (or equivalently, a group of buyers and sellers) has been the focus of important recent studies.

In a recent paper, Gerlagh and Liski (2011) consider a setting in which the seller of a nonrenewable resource faces demand from a buyer who has a substitute with a time-to-build delay. They show that in this simple framework, textbook Hotelling (1931) results are reversed - the stock of the nonrenewable resource decreases over time but the quantity supplied increases steadily up to the point where the buyer decides to switch to the substitute source of energy. The seller of oil keeps increasing the supply over time in order to compensate the buyer for the rising scarcity of the resource, thereby ensuring that the buyer is indifferent between (a) investing in the substitute and (b) buying oil and postponing the investment decision.

In this paper we extend the above framework of Gerlagh and Liski (henceforth called GL) to include learning-by-doing in the substitute. This is an important feature of almost all infant industries - unit costs decline for a significant period of time during the operating life of a technology. In the case of energy, there is a large body of literature which has documented significant cost declines in alternative energy markets such as solar, wind and

[1] President Barack Obama emphasized this argument in his 2013 State of the Union address, "The natural gas boom has led to cleaner power and greater energy independence." (See https://obamawhitehouse.archives.gov/blog/2013/02/13/president-obamas-2013-state-union).

biofuels, as well as in energy technologies used in earlier centuries such as lighting.[2] For example, the use of solar energy in transportation - cars and buses running on electricity powered by solar photovoltaic cells - may be considered an alternative to the use of crude oil. The unit cost of solar has declined steadily with the addition of new capital stock (solar plants). As old plants are retired or become obsolete, they are replaced by new ones; and this brings down the unit cost of producing renewable energy.[3] Other studies have found significant learning effects in solar and wind generation (e.g., McDonald and Schrattenholzer (2001), Duke and Kammen (1999)). Estimates show that the cost of power generation from wind energy has declined from nearly $150/MW to about $50/MW, and is expected to decline even further (Lantz, Hand and Wiser, 2012). Other fuels (such as biofuels) that are direct substitutes for oil in the transportation sector have also experienced cost reductions in recent years with the adoption of high-yielding varieties of corn and more efficient methods of production and refining.[4] After several years of receiving federal subsidies for biofuels which have since been withdrawn, the US is now the world's largest exporter of biofuels, suggesting that some learning may have taken place during the initial period of growth in this industry.

The surprising result in our paper is that when the alternative energy source exhibits significant cost reductions because of learning, the Gerlagh-Liski results disappear. Resource prices rise and quantity declines over time, exactly as Hotelling (1931) predicted. When oil is abundant, the equilibrium

[2]See Fouquet (2006) and Nordhaus (1996).

[3]Solar panels are subject to Swanson's Law, similar to Moore's Law for transistors, which suggests that the cost of photovoltaic cells needed to generate solar power falls by 20 percent with each doubling of global manufacturing capacity. The cost of these cells has dropped two orders of magnitude in the last 35 years, see *http://kottke.org/13/06/alternative-energy-costs-are-dropping*. In California the price of solar panels has dropped from $4/watt in 2007 to under $1/watt, according to Borenstein (2013).

[4]Chen and Khanna (2012) find that processing costs for corn ethanol in the US have declined 17 percent as production volumes increased 17-fold during the period 1983-2005.

price of oil may lie strictly below the buyer's reservation price for alternative energy, generating a surplus for the buyer. When oil becomes scarce, its price equals the true cost of the alternative energy, and there is no buyer surplus. Moreover, if learning is significant, oil may never be used, even though it is cheaper to extract than the substitute source of energy. These results are robust to including a time-to-build delay in developing the substitute, along the lines of Gerlagh-Liski.

We consider a bilateral monopoly where a buyer —or a group of buyers who coordinate their actions– import a nonrenewable resource (say, oil) from a seller or a group of sellers that form a cartel —imagine Western nations buying crude oil from OPEC or natural gas from Russia. The buyer can invest in a substitute that is costlier than oil. However, and this is the key innovation in this paper, the unit cost of the substitute decreases with cumulative use.

Our results on the effects of learning by doing rely on the fact that the substitute to oil is readily available (possibly at an extremely high cost) whenever the buyer decides to switch.[5] This assumption, which differs from Gerlagh and Liski's assumption of a delay between the time of the decision to switch and the time at which the substitute becomes available, is the reason why oil supply does not increase in our model. Extraction decreases until the oil price reaches the net cost of producing solar energy.

Section 2 develops the basic model of buyer and a s seller of a nonrenewable resource with learning in the substitute resource available to the buyer. Section 3 derives the socially optimal solution for resource use and prices. Section 4 models strategic behavior between the buyer and seller. Section 5

[5]We do not model the research and development process that makes the substitute available to the buyer. Many papers have studied optimal investment in research and development for an oil-importing country: see for example Dasgupta *et al.* (1983), Harris and Vickers (1995), Bahel (2011), although not in the strategic context we examine in this paper. Like GL, our analysis focuses on strategic considerations after the backstop becomes available to the buyer, that is, after a successful research and development process has been completed.

concludes the paper. All proofs are relegated to the Appendix.

2 The Buyer-Seller Model with Learning in the Substitute

We propose a simple model along the lines of Gerlagh and Liski (2011), but with learning by doing in the substitute resource. The basic idea is that a producer of oil sells to a buyer who has the option of investing in a substitute resource which exhibits decreasing unit cost with use.

Let x and y be the quantity of oil and solar energy consumed by the buyer at any given time. We hide the time subscript whenever it is convenient and the context is clear. Then the total energy consumed is given by $q = x + y$. Through appropriate normalization, we choose units such that one unit of solar panel is energy equivalent to a barrel of oil. This choice of units is convenient and does not affect our results. Let the buyer's surplus associated with the consumption of energy be given by $u(q)$, which is assumed to be increasing and concave in q. We make the usual assumption that $\lim_{q\to 0} u'(0) = \infty$, which guarantees that the buyer will always consume a positive quantity of energy.[6]

First consider the static relationship between the buyer and seller in the simple case when the buyer only consumes oil. When faced with an oil price p, the buyer chooses the quantity of oil consumed by maximizing his utility given by $u(x) - px$ which yields the necessary condition $p = u'(x)$. Let us write the inverse of this (static) demand as $x(p) = u^{-1}(p)$. The corresponding profit function of the seller can then be written as $\pi(x) = (u'(x) - c)x$, where c is the seller's unit cost of extraction for oil, assumed to be constant.[7] Let p^m denote the seller's monopoly price which satisfies the necessary condition

[6]This assumption enables us to focus only on the interior solution.

[7]Let us assume that the seller's profit function $\pi(x)$ is strictly concave and continuously differentiable in x.

$[u''(x)x + u'(x)] = c$.

We now extend this model to include the dynamic interaction when the buyer has the option to invest in a substitute, which we call solar energy for convenience.[8] Let $p(t)$ be the price set by the seller at date t. When oil is the buyer's only source of energy, his instantaneous surplus at time t is $u(x) - px$; and the seller's profit is $\pi = px - cx$. Let the unit cost of solar energy be defined by $k(Y)$, where Y denotes the cumulative stock of solar panels that have been produced previously. That is, $Y(t) = Y(0) + \int_0^t y(\tau)d\tau$, where $Y(0)$ is the initial endowment of solar panels. The higher the number of panels produced, the lower the unit cost of production of solar energy, $k'(Y) < 0$. Let us assume that these cost reductions decline with the number of panels, $k"(Y) > 0$. That is, learning reduces costs but at a declining rate. The stock of solar panels is a proxy for the know-how accumulated by the buyer in producing solar energy. We abstract from considering market structure in the solar technology and treat it as a competitive industry, as has been done by GL and others in the literature. By normalizing the initial stock of solar panels $Y(0)$ to zero, we can write $k(0) > c$, i.e., the initial unit cost of solar exceeds the cost of oil.[9] If solar energy were cheaper throughout, oil would never be used. Suppose that the lower bound on the unit cost of solar energy is higher than the unit cost of oil, i.e., $\lim_{Y\to\infty} k(Y) = \underline{k}$. Thus by assumption, $c < \underline{k} < k(0)$. Later, we discuss the case where the limit cost of solar energy is lower than the cost of oil, that is, $\underline{k} < c < k(0)$.

If solar energy and oil are consumed simultaneously, the buyer's surplus is given by $u(q) - px - k(Y)y$. The seller's profit can be written as $\pi = px - cx$. Let $\Pi(X_0, Y_0)$ denote the discounted sum of the seller's profit beginning from

[8] This may be any resource that is a substitute for oil in its primary use in transportation, such as biofuels from corn or liquefied shale gas that is used to generate electricity for automobiles.

[9] A typical specification for this learning process may be written as $k(Y) = \theta e^{-\delta Y} + \omega$, where the parameters θ, ω, δ are positive and satisfy $k(0) = \theta + \omega > c$. Here the difference between the unit cost and ω (its limit as Y goes to infinity) decreases at the constant rate $\delta > 0$.

the initial date t_0, given the endowment of oil (X_0) and solar panels (Y_0). Then we can write

$$\Pi(X_0, Y_0) = \int_{t_0}^{\infty} e^{-rt}\pi(t)dt, \tag{1}$$

where r is the discount rate, taken as given and assumed to be equal for both buyer and seller. In an analogous fashion, the intertemporal utility of the buyer, denoted by $W(X_0, Y_0)$, can be written as

$$W(X_0, Y_0) = \int_{t_0}^{\infty} e^{-rt}[u(q) - px - k(Y)y]dt. \tag{2}$$

We write $w(Y_0) \equiv W(0, Y_0)$ to refer to the buyer's intertemporal utility after oil is exhausted.

3 Socially efficient use of oil and solar energy

Assume that the consumer owns the stock of crude oil. Then he can choose the optimal energy mix (of oil and solar energy) that maximizes aggregate utility as follows:

$$W(X_0, Y_0) = \max_{\{x,y\}} \int_0^{\infty} e^{-rt}\left[u(x + y) - cx - k(Y)y\right]dt \tag{3}$$

subject to

$$\dot{X}(t) = -x \qquad \text{(with } X_0 \geq 0\text{)} \tag{4}$$

$$\dot{Y}(t) = y \qquad \text{(with } Y_0 \geq 0\text{)} \tag{5}$$

$$x, y \geq 0 \qquad \text{and } \lim_{t\to\infty} X(t) \geq 0. \tag{6}$$

Conditions (4) and (5) suggest that the stock of crude oil $X(t)$ is depleted as oil is consumed and the stock of solar panels $Y(t)$ increases when more panels are produced. We assume no depreciation of solar panels: adding a constant depreciation rate would add another term to (5) and make the model more complicated to solve without really adding any new insights on the main question we pose in this paper. We can thus write the corresponding current value Hamiltonian as

$$H = u(x+y) - cx - k(Y)y - \lambda x + \beta y, \tag{7}$$

where λ and β are the respective shadow prices attached to the stock of oil and solar panels, respectively. The necessary conditions for maximization are:

$$u'(x+y) \leq c + \lambda \ (= \text{if } x > 0); \tag{8}$$

$$u'(x+y) \leq k(Y) - \beta \ (= \text{if } y > 0); \tag{9}$$

$$\dot{\lambda}(t) = r\lambda; \tag{10}$$

$$\dot{\beta}(t) = r\beta + k'(Y)y; \tag{11}$$

$$\lim_{t\to\infty} \lambda(t)X(t) = 0 \text{ and } \lim_{t\to\infty} \beta(t) = 0. \tag{12}$$

From (8), the marginal value of oil equals the sum of its unit cost and scarcity rent. Equation (9) states that the marginal value of solar energy equals its unit cost minus a subsidy given by β that represents the benefit from learning by doing. Condition (10) gives the usual Hotelling rule - the scarcity rent of oil increases at the rate of discount. Let us define λ_0 as the shadow price of oil at time t_0. Then $\lambda(t) = \lambda_0 e^{rt}$. Note that the subsidy on solar energy may increase or decrease over time depending on the net effect of the two terms in (11) since the second term is negative. Let β_0 be the value of the subsidy β at time t_0. Condition (12) essentially says that aggregate oil extraction cannot exceed the initial stock of oil X_0: if oil is exhausted we will have $\lim_{t\to\infty} X(t) = 0$; otherwise, $\lim_{t\to\infty} \lambda(t) = 0$. Since the Hamiltonian is concave in its arguments (x, y, X, Y, t), the above necessary conditions are also sufficient for optimality. We can now state the following result.

Proposition 1 *The socially optimal solution implies that (a) if initially, oil is cheaper than solar energy by less than some positive* β_0 *(where* $k(Y_0) - c < \beta_0$*) then oil will never be extracted - only solar energy will be used from the beginning; (b) otherwise, oil is used exclusively for some time until a complete transition to solar energy.*

Proof: See Appendix

Note that the price of crude oil must go up over time due to scarcity. However, solar energy prices go down because of learning by doing. Because the price of crude oil is increasing and the price of solar is decreasing, there is no simultaneous use of the two resources over any (non-degenerate) time interval. Therefore, only two outcomes are possible. When solar is cheap relative to oil and the cost reductions from using solar are high (reflected in a high value of the subsidy β), the social planner will use only solar from the beginning, and *no oil will ever be used.* This may happen even if oil is initially cheaper than solar energy.

If oil is cheap or abundant, or if the learning effect of solar is relatively small, we first use oil and then switch at some point in time to solar energy. Note that aggregate energy consumption decreases when the fossil fuel is supplied and increases when solar panels are deployed. In the limit, the price of energy approaches the limiting marginal cost of solar energy $\underline{k}$. Energy prices first rise and then fall when oil is used, and consumption follows a U-shaped path as shown in Figure 4, where oil is used until time T, after which it is replaced by solar energy.[10]

[10] As shown in the Appendix, the solution is uniquely determined by the following system of two equations with two unknowns, λ and T:

$$\begin{cases} (\beta + \lambda_0)e^{rT} = k(0) - c \\ \int_0^T \tilde{x}(c + \lambda_0 e^{rt})dt = X_0. \end{cases} \tag{13}$$

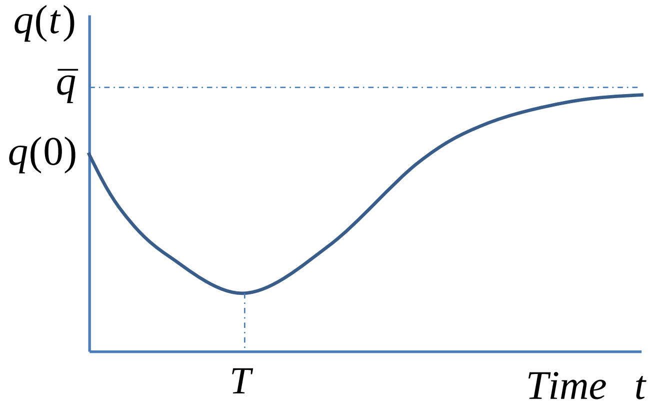

Figure 1: Aggregate energy consumption falls and then rises. Only oil is consumed until time $\bar{T}$ when it is replaced by solar energy.

4 Strategic Interaction between Buyer and Seller

We can now study the differential game between the buyer and seller. We treat time as continuous and consider small intervals of the form $[t, t + dt]$. We let the two agents make their respective decisions at the beginning of each interval and commit to those choices for the entire period of length dt. Until oil is exhausted, the sequence of decision-making in each interval $[t, t+dt]$ is as follows:

(1) Given the stocks of oil $X(t)$ and solar panels $Y(t)$, the seller chooses a unit price $p(t)$ for oil.[11]

(2) The buyer then chooses $x(t)$ and $y(t)$, her respective consumption bundles of oil and solar panels.

Recall that after exhaustion of the oil stock, the buyer relies only on solar energy and her discounted utility is given by $w(Y_0) \equiv W(0, Y_0)$. In what follows, we determine the Markov perfect equilibrium of the dynamic

[11] The price-setting approach we adopt is similar to that of GL who consider a quantity-setting seller. With learning by doing, it is convenient to let the seller set a price and the buyer choose quantity.

bilateral monopoly described above when dt approaches zero.

Conversely, we assume that the buyer finds it economical to use some crude oil and not leave the entire stock in the ground, so that strategic interaction can occur. That is, $c < k(Y_0) - \beta$, oil is cheaper than the subsidized price of solar energy. This assumption is reasonable, as many studies have concluded that the cost of fossil fuels may be lower than the true social cost of cleaner energy sources such as solar or wind energy (e.g., see Borenstein, 2012).

Let $p(t)$ be the price offered to the buyer by the seller. At any date t_0 the buyer's problem can be written as:

$$W(X_0, Y_0) = \max_{\{x,y\}} \int_{t_0}^{\infty} e^{-r(t-t_0)}[u(x+y) - p(X,Y)x - k(Y)y]dt \tag{14}$$

subject to (4) and (5).

In (14), unlike in the socially optimal case, the buyer acquires barrels of oil at the seller's unit price $p(t)$, which may not be the same as the unit cost of oil c. Thus the structure of the problem remains the same and the results are qualitatively the same as stated in Proposition 1, except that c is now replaced by the buyer's price $p(t)$. The buyer will buy oil if it is cheaper than the cost of solar net of the benefits from learning, in other words, if $p < c - \alpha$. We can now state the following:

Proposition 2 *The buyer's Markov perfect strategy is as follows: (a) when oil is abundant (higher than some threshold stock $\bar{X}$), the buyer is "myopic" - he consumes oil if and only if $k(Y) - \alpha \geq p$ (and zero otherwise); (b) when oil is scarce - lower than the threshold stock $\bar{X}$, the buyer imposes a tax μ on his own consumption and consumes a lower quantity of oil, i.e., $x = x(p+\mu)$ iff $k(Y) - \alpha \geq p$, and none otherwise.*

Proof: See Appendix

What is surprising here is that when the stock of oil goes below this threshold level, the buyer self-imposes a "tax" on oil to *curb* his own consumption. He does not behave like a myopic buyer. This is because by doing so, he enjoys a positive surplus as long as the oil stock remains above a given level. This is due to the fact that the seller prices oil at lower than the net cost of solar energy as long as the oil stock remains relatively large, as will become more transparent below when we discuss the seller's strategy.

The seller's objective is to maximize the discounted sum of his instantaneous profits $\pi = (p - c)x$, given the optimal purchase strategy of the buyer:

$$\Pi(X(0), Y(0))) = \max_{p(t)} \int_{t_0}^{\infty} e^{-r(t-t_0)} (p - c)\, x dt \tag{15}$$

subject to:

$$\dot{X}(t) = -x$$
$$p \leq k(Y) - \alpha.$$

We need the last condition because the buyer does not buy oil if the net price of solar energy is lower than the oil price, i.e., when $p > k(Y) - \alpha$. Recall that p^m denotes the (static) monopoly price of the seller. We can now describe the seller's pricing strategy as follows:

Lemma 1 *At any instant, the seller's optimal price $p(X, Y)$ satisfies:*

$$\begin{cases} p(X,Y) \in [p^m, k(Y) - \alpha], & \textit{if } p^m \leq k(Y) - \alpha; \\ p(X,Y) = k(Y) - \alpha, & \textit{otherwise.} \end{cases}$$

Note that the seller will never price above the net marginal cost (which is unit cost net of subsidy or $k(Y) - \alpha$) at which the buyer can produce the substitute because then the buyer will not use oil. If the seller's static monopoly price is lower than the net cost of the substitute to the buyer, then the closer is the seller's price to the monopoly price, the higher are his profits. However, the seller may want to price higher than the monopoly price, simply to spread his profits over a longer time period.

The seller's complete strategy can be summarized by the following:[12]

Proposition 3 *The Markov perfect pricing strategy of the seller is: (a) When the monopoly price is lower than the net cost of solar energy. Then if oil is abundant, the seller charges a price strictly below the net price of solar energy to the buyer. If oil is scarce (the stock falls below a threshold level), the seller charges a price exactly equal to the net cost of solar energy; (b) when the monopoly price is higher than the net cost of solar energy, the price charged equals the net price of solar energy.*

Intuitively, if oil was not scarce at all (when the stock is infinite), the seller would charge the monopoly price p^m to the buyer at each date, provided p^m is lower than the reservation price of the buyer. Since the stock is actually finite, the seller will optimally charge a price that is higher than p^m.[13] Furthermore, due to discounting, the seller adopts an increasing price path, so that he can earn higher profits in earlier periods. This explains why the oil price is lower than the buyer's reservation price while oil is still relatively abundant.

Thus, if oil is abundant and the monopoly price is lower than the net cost of solar energy, the buyer gets a surplus and reduces his consumption of oil, to ensure that the stock of oil lasts for as long as possible. This is different from Gerlagh and Liski where the buyer gets no surplus and does not worry about depletion. Figure 4 shows the evolution of the oil price and consumption over time.

[12]We show in the Appendix that the Markov perfect equilibrium described by Propositions 2 and 3 is unique if we impose the following conditions: (a) the pricing strategy of the seller $p(X,Y)$ is a non-decreasing function of the oil stock X for any fixed stock of solar panels Y; (b) the purchasing strategy of the buyer $x(X,Y,p)$ is a decreasing function of the price p, for any given X and Y.

[13]Indeed, a price lower than p^m would not only reduce current profits, but also leave the seller with a lower stock for the future.

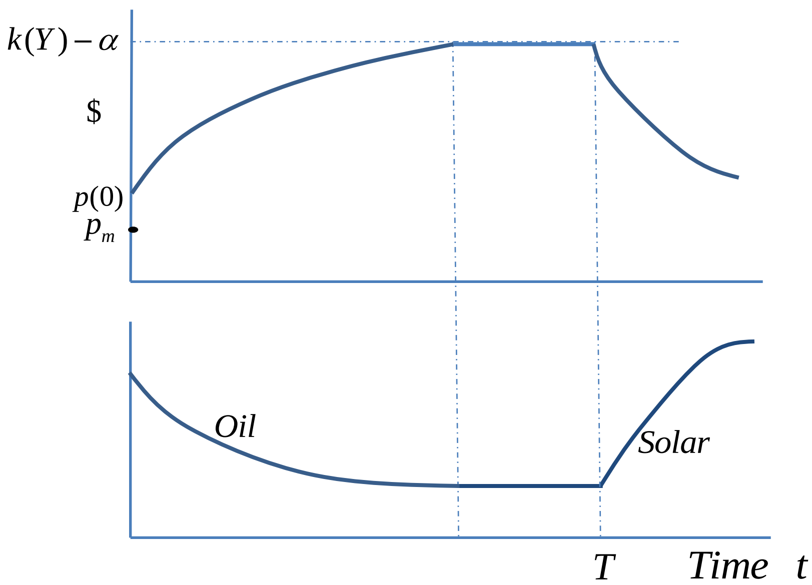

Figure 2: Oil price rises over time, stays below the net cost of solar energy and approaches the net price of solar at time T.

The buyer initially prices oil below the reservation price of the seller (net price of the clean substitute) until the threshold stock is reached. After that, the price stays constant at the level of the substitute. The higher the initial stock of oil, the longer is the first phase of low oil prices. Note that in the unlikely case that the monopoly price is higher than the net cost of the substitute, the price will always equal the substitute price, and the first stage disappears. When the stock of oil is depleted, the buyer switches to solar energy and consumption increases to its asymptotic limit.

If learning has a significant effect on costs, then the subsidy β is likely to be large, so that $k(0) - \beta < c < k(0)$, the buyer adopts solar energy from the outset, even if its unit cost is higher than the unit cost of oil.

5 Concluding Remarks

Our analysis incorporates learning-by-doing into the strategic resource dependence model. We have considered a nonrenewable resource bilateral monopoly where the buyer has the option to adopt a substitute whose marginal cost decreases as cumulative use increases. The results exhibit interesting qualitative differences in comparison with the findings of Gerlagh and Liski (2011). In particular, we show that the oil stock may be discarded even with a substitute that is much costlier than oil.

We find that the Markov-perfect outcome exhibits a stage where the seller prices oil below the reservation price and the buyer curbs oil consumption in order to enjoy a positive surplus for as long as possible. Indeed, the buyer conserves oil because, on the equilibrium path, the seller sets the price equal to the buyer's reservation price as soon as the stock falls below a given threshold. As far as we know, this "non-myopic" behavior of the buyer (who is concerned with the depletion of the stock) is new to the literature on strategic resource dependence. In the Markov perfect equilibria of Gerlagh and Liski (2011), the buyer is essentially indifferent between (a) investing immediately and (b) consuming oil while postponing the adoption of the substitute.

For simplicity and comparison purposes,[14] we have considered a unit cost for the substitute that only changes with the aggregate stock of supply of panels, but is constant at any given date. For example, the marginal cost does not vary with the number of panels produced instantaneously. As a consequence, the buyer consumes either oil or solar energy (but not both) at any given time. We leave for future research the case of a learning-by-doing process with a nonlinear cost function for the substitute production.[15] We believe such a framework would yield much of the same results as our present

[14] Note that Gerlagh and Liski (2011) assume a constant unit cost for the substitute.

[15] Chakravorty, Leach and Moreaux (2012) examine such a learning process with nonlinear costs. They show that the combined effects of learning-by-doing and environmental regulation may lead to alternate periods of rising and falling oil prices.

paper, while allowing the simultaneous use of the different sources of energy.

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A Proofs

In this Appendix, we provide the proofs of the results stated in the text, as well as some additional results in Propositions 4-5. Propositions 1-3 are stated in non-technical language in the main text, and here we provide a technical version before offering a proof. Proposition 1 relates to the socially efficient extraction path. Propositions 2-3 relate to the description of the Markov equilibrium between buyer and seller. Finally, Propositions 4-5 give an analytical description of the socially efficient path.

The following lemma states that the buyer will no longer purchase oil once solar energy is used. This result is useful in proving Proposition 1, which describes the socially efficient use of oil and solar energy.

Lemma 2 *The solution* $(x(t), y(t))_{t\geq 0}$ *to (3) is such that: for any* $t_1 \geq 0$*,*

$$y(t_1) > 0 \Rightarrow (x(t) = 0, \text{ for all } t > t_1).$$

Proof: Suppose that $y(t_1) > 0$ for some $t_1 \geq 0$. Then the necessary condition (9) implies that

$$u'(x(t_1) + y(t_1)) = k(Y(t_1)) - \beta(t_1)e^{rt_1}.$$

Combining this with (8), one can write

$$k(Y(t_1)) - \beta(t_1)e^{rt_1} = u'(x(t_1) + y(t_1)) \leq c + \lambda(t_1)e^{rt_1}.$$

In other words, at date $t = t_1$, the current value of the full marginal cost of solar energy , $k(Y) - \beta(t)e^{rt}$, is no higher than that of oil consumption, $c + \lambda(t)e^{rt}$. Note in addition that, since $\dot{\lambda}_t^o = 0$ [by condition (10)], the full marginal cost of oil $(c + \lambda(t)e^{rt})$ is nondecreasing over time.

In order to conclude the proof, it is thus sufficient to show that $k(Y) - \beta(t)e^{rt}$ is decreasing (over time) on the optimal path. To that end, let us observe that:

$$\frac{d\left[k(Y)-\beta(t)e^{rt}\right]}{dt} = k'(Y)\dot{Y} - \dot{\beta}(t)e^{rt} - r\beta(t)e^{rt}$$
$$= k'(Y)y - \dot{\beta}(t)e^{rt} - r\beta(t)e^{rt} \qquad \text{by the law of motion (5).}$$

Using the necessary condition (10), we can write $k'(Y)y = \dot{\beta}(t)e^{rt}$. This, plugged into the above time derivative, gives:[16]

$$\frac{d\left[k(Y)-\beta(t)e^{rt}\right]}{dt} = \dot{\beta}(t)e^{rt} - \dot{\beta}(t)e^{rt} - r\beta(t)e^{rt} = -r\beta(t)e^{rt} < 0.$$

This shows that $k(Y) - \beta(t)e^{rt} < c + \lambda(t)e^{rt}$ for any $t > t_1$. That is to say, if solar energy is used at date t_1, oil will not be used from t_1 onwards (due to its higher full marginal cost).□

The following proposition is the technical version of Proposition 1 in the text. It describes the characteristics of efficient oil and solar power usage.

Proposition 1 *(technical version)*

On the socially optimal path, we have the following.

(i) *Given* $k(\cdot)$ *—the learning process, there exists a unique threshold* $\alpha \in (0, k(0))$ *such that:* a. *if* $k(0) - c < \alpha$ *then* $x(t) = 0$*, for any* $t \geq 0$*; and* b. $x(0) > 0$ *if* $k(0) - c > \alpha$*. (ii) There exists a date* $T \geq 0$ *such that* $y(t) = 0$*,* $y(t) > 0$*, for* $t \geq T$*; and* $y(t) > 0$*,* $x(t) = 0$*, for* $t > T$*.*

Proof: By (10), we have $\lambda(t) = \bar{\lambda} \geq 0$, for any $t \geq 0$. Let then $\beta(0) \equiv \alpha$ be the optimal shadow value of solar energy at date $t = 0$. At $t = 0$, the respective full marginal costs for solar energy and oil are $k(0) - \alpha$ and $c + \bar{\lambda}$.

(i).a If $\underline{k(0) - c < \alpha}$, then we have $k(0) - \alpha < c + \bar{\lambda}$. In this case, combining the conditions (9) and (10), shows that $q^s_{t=0} > 0$ and $q^o_{t=0} = 0$. This, given the result of Lemma 2, proves the first claim (a).

[16]Notice that the shadow value $\beta(t)$ is positive (and decreasing) by condition (10), since $k'(Y) < 0$.

(i).b If instead $\underline{k(0) - c > \alpha}$: by way of contradiction, suppose that $q^o_{t=0} = 0$ —this means that, $k(0) - \alpha < c + \bar{\lambda}$ and $q^s_{t=0} > 0$. It then follows from Lemma 2 that $q^o_t = 0$, $\forall t > 0$ (thus, oil is never used and $X = X_0$ for any t). In addition, the transversality condition ($\lim_{\infty} \lambda(t) X_0 = 0$) implies that $\lambda(t)$=0. Hence, $k(0) - \alpha > c + \bar{\lambda} = c$, which is a contradiction. Therefore, we must have $q^o_{t=0} > 0$ if $k(0) - c > \alpha$.

(ii) If $k(0) - c < \alpha$ then it suffices to take $T = 0$ [by (i).a]. Otherwise, it follows from the case (i).b above that $k(0) - \alpha > c + \bar{\lambda}$ and $q^o_{t=0} > 0$ (while $q^s_{t=0} = 0$). For any T satisfying $q^s_t = 0$ $\forall t \in [0, T]$, we get from the (5) that $Y = 0$ $\forall t \in [0, T]$. It then follows from (10) that $\beta(t) = \lambda^s_{t=0} = \alpha$ $\forall t \in [0, T]$.

Thus, as long as $q^s_t = 0$, the full marginal cost of solar energy (in current value) is $k(0) - \alpha e^{rt}$ and decreases over time. On the other hand, the full marginal cost of oil is given by $c + \bar{\lambda} e^{rt}$ and increases over time. It is easy to see that $c + \bar{\lambda} e^{rt} = k(0) - \alpha e^{rt}$ for $t = T = \ln\left(\frac{k(0)-c}{\bar{\lambda}+\alpha}\right)$; and $c + \bar{\lambda} e^{rt} > k(0) - \alpha e^{rt}$ for $t > T$. Therefore, solar energy is used after T (due to its lower full marginal cost) and, from Lemma 2, oil is not consumed from T on (that is, $q^o_{T+t'} = 0$).$\square$

A.1 Proposition 2

We now give a technical and explicit version of Proposition 2 in the text, which dscribes the buyer's Markov strategy.

Proposition 2 *(technical)*

There exists a Markov-perfect strategy for the buyer; it is characterized by two functions $\tilde{X} \equiv \tilde{X}(Y)$ and $\mu \equiv \mu(X, Y)$ s.t.

- **a-** whenever $X \leq \tilde{X}$: $q_t = x(t) = \tilde{x}(p_t)$ if $k(Y) - \alpha \geq p_t$ (with $x(t) = 0$ otherwise);
- **b-** whenever $X > \tilde{X}$: $q_t = x(t) = \tilde{x}(p_t + \mu)$ if $k(Y) - \alpha \geq p_t$ (with $x(t) = 0$ otherwise).

Proof: For the buyer, the cost of acquiring a barrel of oil at any date t is p_t, and the full marginal cost of the substitute is $k(Y) - \alpha$. It is straightforward to see from Section 3 that the buyer optimally consumes solar energy (that is, $x(t) = 0$) at any time t such that $p_t > k(Y) - \alpha$.

In the case where $p_t \leq k(Y) - \alpha$, we have $y(t) = 0$ and $x(t) > 0$ instead. It will be shown in the proof of Proposition 3 that (in the Markov-perfect equilibrium) the seller prices above the buyer's reservation price $k(Y) - \alpha$ whenever the oil stock X is above some threshold $\tilde{X}(Y)$. And for any $X \leq \tilde{X}(Y)$, the seller will choose $p_t = \bar{p} = k(Y) - \alpha$. Thus, the oil stock has no value for the buyer as soon as $X \leq \tilde{X}(Y)$ —since he is charged the same price from then on; and he behaves in a myopic way by consuming $\tilde{x}(p_t)$. On the other hand, the buyer receives a positive surplus [above the reservation payoff $\bar{u} \equiv u\left(\tilde{x}(\bar{p})\right)$] as long as $X > \tilde{X}(Y)$. Thus, given the Markov strategy of the seller, $p(X)$, the buyer has to solve the following problem at any date t [s.t. the oil stock is $X > \tilde{X}(Y)$]:

$$\max_{\{q_\tau\}_{t \leq \tau \leq \bar{T}}} \int_t^{\bar{T}} e^{-r\tau} \left[u(q_\tau) - p(X_\tau)q_\tau - \bar{u}\right] d\tau \tag{16}$$

subject to:

$$\dot{X}_\tau = -q_\tau \text{ and } X_{\bar{T}} = \tilde{X}(Y).$$

Writing the Hamiltonian and the optimality conditions for this problem, it is easy to see that the buyer's optimal oil consumption at date t assumes the form $x = \tilde{x}(p_t + \mu)$, where $p_t \equiv p(X)$ and $\mu \equiv \mu(X, Y)$ is the oil scarcity value [associated with the problem (16)] at date $\tau = t$.□

A.2 Lemma 3

The following lemma is a useful step in describing the seller's equilibrium strategy. It gives the range of the oil price.

Lemma 3 *At any date t such that $k(Y) - \alpha > c$, the seller's optimal price $p(X,Y)$ satisfies:*

$$\begin{cases} p(X,Y) \in [p^m, k(Y) - \alpha], & \text{if } p^m \leq k(Y) - \alpha; \\ p(X,Y) = k(Y) - \alpha, & \text{otherwise.} \end{cases}$$

Proof: At any date t, a price $p_t > k(Y) - \alpha$ would trigger the buyer's use of the substitute (instead of oil) at date t, that is to say, $y > 0 = x$. This would have two negative effects on the seller: (a) his profit at date t would be zero; (b) his continuation value $W(X,Y)$ would decrease due to a higher L after t.[17] It is clearly better for the seller to choose

$$p_t \leq k(Y) - \alpha \tag{17}$$

and make positive profits at date t while preventing the know-how of the buyer (L) from increasing. Therefore, in a Markov-perfect equilibrium, we will always have $p_t \leq k(Y) - \alpha$. Recall that p^m is the optimal price of the seller for the (static) demand function $D(p)$. One can see that the instantaneous profit $\pi_t = (p_t - c)D(p_t)$ is increasing in p_t as long as $p_t < p^m$. Let us now discuss the following two cases.

• Suppose that $p^m \leq k(Y) - \alpha$ at date t. By choosing a price $p_t < p^m$, the seller earns lower profits at t (than with a price of p^m) and is left with a lower oil stock due to higher sales at t; this is clearly not optimal. Thus, in a Markov-perfect equilibrium, we have $p^m \leq p_t \leq k(Y) - \alpha$.

• If $p^m > k(Y) - \alpha$, choosing $p_t < k(Y) - \alpha$ would give lower instantaneous profits while depleting the stock faster (which is not optimal). This shows that we must have $p_t \geq k(Y) - \alpha$. Recalling (17) then gives the desired result: $p_t = k(Y) - \alpha$.

[17] Note that L increases because $y > 0$ and $\dot{Y} = y$. Furthermore, recall that the cost $k(L)$ of producing solar energy is a decreasing function of L.

A.3 Proposition 3

We are now set to fully describe the seller's strategy. The following is the technical version of Proposition 3 in the text

Proposition 3 *(technical)*

There exists a Markov-perfect pricing strategy of the seller s.t.

(i). $p(X,Y) = k(Y) - \alpha$ if $p^m \geq k(Y) - \alpha \geq c$;

(ii). if instead $p^m < k(Y) - \alpha$ then

a- $p(X,Y) = k(Y) - \alpha$ for $X \leq \tilde{X}$;

b- $p(X,Y) = u'(\pi'^{-1}(e^{r(t-T_1)}\pi'(\bar{x}))) - \mu$ for $X > \tilde{X}$, where $T_1 \equiv T_1(Y,X)$.

Proof: We show that the pricing strategy described in Proposition 3 is the seller's best response against the buyer's strategy of Proposition 2 (and vice-versa).

We first show that, against any strategy of the buyer which can be written in the form $\tilde{q}^o(X,Y,p_t) = \tilde{x}(p_t + \mu)$ [where $\mu \equiv \mu(X,Y)$],[18] the seller's best response involves a threshold $\tilde{X}$ such that: $p_t = \bar{p} \equiv k(Y) - \alpha$ if $X < \tilde{X}$; and $p_t < \bar{p}$ otherwise. Indeed, for any such strategy of the buyer, the problem (15) of the seller at date t_0 can be written as:[19]

$$\max_{T_1,\{x\}_{t\in[t_0,T_1]}} \int_{t_0}^{T_1} e^{-r(t-t_0)} \left(p_t - c - \mu\right) x dt + e^{-r(T_1-t_0)} \left[1 - e^{-r\frac{X_{T_1}}{\bar{x}}}\right] (\bar{p} - c)\bar{x}/r \quad (18)$$

subject to:

$$\dot{X} = -x \text{ (with } X_{t_0} = X_0)$$
$$x(T_1) = \bar{x} \equiv \tilde{x}(k(Y_{t_0}) - \alpha).$$

[18] Note that the corresponding inverse demand function is $p_t = u'(x) - \mu$ when the seller chooses oil extraction x.

[19] Note that we are implicitly using Lemma 3 to write the second term of the objective (18): when the price reaches $\bar{p}$ at date T_1, it can no longer increase and will remain constant at $\bar{p}$ for the time period $[T, T + \frac{X_T}{\bar{x}}]$. Equivalently, oil supply increases up until T and then remains constant at $\bar{x}$ during the period $\bar{x}$ during the period $[T_1, T_1 + \frac{X_{T_1}}{\bar{x}}]$.

Recalling the function π and letting $\tilde{\mu} \equiv e^{-r(t-t_0)}\mu$, one can rewrite the objective:

$$\max_{T_1, \{x\}_{t\in[t_0,T_1]}} \int_{t_0}^{T_1} \left[e^{-r(t-t_0)}\pi(x) - \tilde{\mu}x\right] dt + e^{-r(T_1-t_0)} \underbrace{\left[1 - e^{-r\frac{X_{T_1}}{\bar{x}}}\right] \pi(\bar{x})/r}_{B(X_{T_1})} .$$

Thus, the Hamiltonian is: $H = e^{-r(t-t_0)}\pi(x) - [\tilde{\mu} + \lambda]\, x$, where λ is the seller's scarcity value for the oil stock. Using Pontryagin's maximum principle, we obtain the optimality conditions:

$$\pi'(x) = e^{r(t-t_0)} \left[\tilde{\mu} + \lambda\right] \tag{19}$$

$$\dot{\lambda} = \frac{\partial \tilde{\mu}}{\partial X} x \tag{20}$$

$$\dot{X} = -x \tag{21}$$

$$H(T_1) = -\frac{\partial \left[e^{-rT_1} B(X_{T_1})\right]}{\partial T_1} \tag{22}$$

$$\lambda(T_1) = -\frac{\partial \left[e^{-rT_1} B(X_{T_1})\right]}{\partial X} \tag{23}$$

Note that (22) is the transversality condition relating to the free terminal time T_1, whereas (23) pertains to the free terminal stock X_{T_1}. We combine these conditions to find $\tilde{X} \equiv X_{T_1}$.

First, observe from (20) that $\dot{\tilde{\mu}} + \dot{\lambda} = \dot{\tilde{\mu}} + \frac{\partial \tilde{\mu}}{\partial X} x = \frac{\partial \tilde{\mu}}{\partial X} \dot{X} + \frac{\partial \tilde{\mu}}{\partial X} x = 0$ —given that $\dot{X} = -x$ by (21). That is to say, $\tilde{\mu} + \lambda$ is constant over time. Plugging this into (22) then gives:

Next, note from (22) that:

$$e^{-r(T_1-t_0)}\pi(x(T_1)) - [\tilde{\mu} + \lambda]\, x(T_1) = e^{-r(T_1-t_0)} \left[1 - e^{-r\frac{X_{T_1}}{\bar{x}}}\right] \pi(\bar{x}).$$

Recalling that $x(T_1) = \bar{x}$ and simplifying, we obtain

$$[\tilde{\mu} + \lambda]\,\bar{x} = e^{-r(T_1-t_0)} e^{-r\frac{X_{T_1}}{\bar{x}}} \pi(\bar{x}). \tag{24}$$

Taking $t = T_1$ in (19), one can write $\tilde{\mu} + \lambda = e^{-r(T_1-t_0)}\pi'(\bar{x})$. Plugging this last equality into (24), we get $e^{-r\frac{X_{T_1}}{\bar{x}}} = \frac{\bar{x}\pi'(\bar{x})}{\pi(\bar{x})}$. It thus follows that

$$\tilde{X} \equiv X_{T_1} = \frac{\bar{x}}{r} \ln\left(\frac{\pi(\bar{x})}{\bar{x}\pi'(\bar{x})}\right).$$

Note that $\tilde{X}$ is a function of Y_{t_0} —just as $\bar{x}$— but does not depend on the initial stock, X_{t_0}. Also remark that the stock X_{T_1} above always exists given our assumption that π is concave (i.e., $\pi(\bar{x}) > \bar{x}\pi'(\bar{x})$). From what precedes, we conclude that $p_t = k(Y) - \alpha$ whenever $X \leq \tilde{X}$ (that is, $T_1 = 0$).

In the case where $X > \tilde{X}$, it follows that $T_1 > 0$. Recalling from above that $\tilde{\mu} + \lambda = e^{-r(T_1-t_0)}\pi'(\bar{x})$, one can use (19) to characterize x:

$$\pi'(x) = e^{r(t-t_0)} e^{-r(T_1-t_0)} \pi'(\bar{x}) = e^{r(t-T_1)}\pi'(\bar{x}).$$

It follows that $x = \pi'^{-1}(e^{r(t-T_1)}\pi'(\bar{x}))$. As stated by Proposition 3, we thus have $p_t = u'(x) - \mu = u'(e^{r(t-T_1)}\pi'(\bar{x})) - \mu$, where T_1 is determined by the condition:[20]

$$\int_0^{T_1} \underbrace{\pi'^{-1}(e^{r(t-T_1)}\pi'(\bar{x}))}_{x}\, dt = X_0 - \tilde{X} \tag{25}$$

and μ is determined by the combination of (19) and the transversality condition (23).□

[20]Equation (25) states the fact that the oil stock is eventually exhausted.

A.4 Proposition 4

The following shows that consumption of oil (solar power) is weakly monotonic.

Proposition 4 *The socially optimal path for oil consumption,* $(x(t))_{t\geq 0}$*, is nonincreasing over time; whereas that of solar energy consumption,* $(y(t))_{t\geq 0}$*, is nondecreasing. In addition, we have* $\lim_{t\to\infty} y(t) = \bar{x}$*, where* $\bar{x} = \tilde{x}\left(\underline{k}\right)$*.*

Proof: The desired result is easily obtained by combining Proposition 1-(ii) and the facts that the full marginal cost of oil $(c + \bar{\lambda}e^{rt})$ is increasing whereas that of solar energy $(k(0) - \beta_t e^{rt})$ is decreasing —as shown in the proof of Lemma 2. Hence, solar energy consumption increases after date T and, given that $\lim_{t\to\infty} Y = +\infty$, condition (9) —with equality— gives $\lim_{t\to\infty} q_t^s = \tilde{x}\left(\lim_{L\to\infty} k(L)\right) \equiv \bar{x}.\square$

A.5 Proposition 5

Accounting for all possible cases (depending on the values of our parameters), the followoing Proposition 5, which is not stated in the text, gives an explicit description of the socially efficient path for oil and solar power usage.

Let us denote by Y_t^* the unique solution to the second-order differential equation (SODE)

$$u''(\dot{Y}_t)\ddot{Y}_t - r\left[u'(\dot{Y}_t) - c_s(Y_t)\right] = 0 \qquad (26)$$

that satisfies the initial and terminal conditions: $Y_{[t=0]} = 0$ and $\lim_{t\to\infty} Y_t = +\infty$. In addition, for any $t \geq 0$, let

$$\beta_t^* \equiv u''(\dot{Y}_t^*)\ddot{Y}_t^*. \qquad (27)$$

Note that $(\beta_t^*)_{t\geq 0}$ is uniquely determined by $r, u(.), c_s(.)$, which are primitives of the model. The upcoming result fully characterizes the socially efficient path.

Proposition 5 *The socially efficient path can be expressed (depending on the relevant case) as follows.*
(i) If $\underline{k(0) - c < \alpha}$*:* $T = 0$*; for any* $t \geq 0$*,* $x(t) = 0$ *and* $q_t = y(t) = \tilde{x}\left(k(Y_t^*) - \beta_t^* e^{rt}\right)$*, where* Y_t^* *and* β_t^* *are respectively given by (26) and (27).*
(ii) If $\underline{k(0) - c \geq \alpha}$*: then there exists a threshold* $\bar{S}$ *such that*

a- *whenever* $X_0 \geq \bar{S}$*, we have* $T = \frac{1}{r} \ln\left(\frac{k(0)-c}{\alpha}\right)$ *and*

$$\begin{cases} q_t = x(t) = \tilde{x}(c), & \text{for } t \leq T \\ q_t = y(t) = \tilde{x}\left(k(Y_{t-T}^*) - \beta_{t-T}^* e^{rt}\right) & \text{for } t > T; \end{cases}$$

b- *whenever* $X_0 < \bar{S}$*, we have* $T = \overline{T}$ *and*

$$\begin{cases} q_t = x(t) = \tilde{x}(c + \bar{\lambda} e^{rt}), & \text{for } t \leq T \\ q_t = y(t) = \tilde{x}\left(k(Y_{t-T}^*) - \beta_{t-T}^* e^{rt}\right) & \text{for } t > T, \end{cases}$$

where $(\bar{\lambda}, \overline{T})$ *is the solution to (13).*

Proof: Let us first determine the optimal solar energy consumption path after the switch to solar energy at date T.[21] The necessary conditions (9)-(11) come down to:

$$u'(y) = k(Y) - \beta_t e^{rt} \quad (28)$$

$$\dot{\beta}_t = e^{-rt} k'(Y) y \quad (29)$$

$$\dot{Y} = y \quad (30)$$

Combining (28)-(30), we obtain the following second-order differential equation:

$$u''(\dot{Y})\ddot{Y}_t - r\left[u'(\dot{Y}) - k(Y)\right] = 0. \quad (31)$$

[21] It is known from Proposition 1 that oil is no longer consumed after the switch.

Let then Y_t^* be the unique solution to (31) that satisfies the initial and terminal conditions $Y^*_{[t=0]} = 0$ and $\lim_{t\to\infty} Y_t^* = +\infty$.[22] The optimal shadow value of solar energy at date $t \geq T$ is then

$$\beta_t^* = u''(\dot{Y})\ddot{Y}_t, \tag{32}$$

as specified in (27). Recall that we defined α as the shadow value of solar energy at $t = 0$ —and as long as $t \leq T$, due to (10). It follows from what precedes that:

$$\beta_t = \begin{cases} \alpha, & \text{if } t \leq T \\ \beta^*_{t-T}, & \text{if } t > T, \end{cases} \tag{33}$$

where β_t^* is given by (32). It is easy to see that β_t is nonincreasing and continuous. We now discuss the different cases of Proposition 5.

(i) Suppose that $\underline{k(0) - c < \alpha}$: then the desired result follows from the combination of Proposition 1-(i), (28) and (33) [where $T = 0$].

(ii) If $\underline{k(0) - c > \alpha}$: then it follows from Proposition 1-(i) that $q_t^o > 0$ (oil is used at the outset). As seen in the proof of Proposition 1, the switch to solar energy is made *by the latest* at $T^* = \ln\left(\frac{k(0)-c}{\alpha}\right)$,[23] regardless of the remaining oil stock X_{T^*}.

a− In the subcase where $X_0 \geq \bar{S} \equiv \tilde{x}(c)T^*$, the optimal oil consumption *up until* T^* is clearly constant and given by $q_t^o = \tilde{x}(c)$ (that is, oil is not scarce and $\bar{\lambda} = 0$). Combining (28) and (33) then gives $y(t) = \tilde{x}\left(k(Y^*_{t-T}) - \beta^*_{t-T}e^{rt}\right)$ for $t > T$.

b− When $X_0 < \bar{S} \equiv \tilde{x}(c)T^*$, the constant oil consumption path above is not feasible (i.e., oil is scarce and $\bar{\lambda} > 0$). Oil consumption is then given by $q_t^o = \tilde{x}(c + \bar{\lambda}e^{rt})$, for any $t \leq T$. And the optimal date T of the switch to solar energy is such that the $k(0) - \alpha e^{rT} = c - \bar{\lambda}e^{rT}$, that is to say, the (current)

[22]Note that Y_t^* is uniquely determined given the primitives of the model: u, k, r.
[23]This is because $\bar{\lambda} \geq 0$.

full marginal costs are equal. It also follows from (12) that $X_T = 0$, which gives: $\int_0^T \tilde{x}\,(c + \lambda e^{rt})\,dt = X_0$.

Combining the two conditions above, one gets the system introduced in (13):

$$\begin{cases} (\alpha + \lambda)e^{rT} = k(0) - c \quad [\text{i.e., } T(\lambda) = \ln\left((k(0) - c)/(\alpha + \lambda)\right)] \\ \int_0^T \tilde{x}\,(c + \lambda e^{rt})\,dt = X_0. \end{cases}$$

Using the intermediate value theorem, it is easy to see that there exists a unique $\bar{\lambda}$ that solves $\int_0^{T(\lambda)} \tilde{x}\,(c + \lambda e^{rt})\,dt = X_0$. Letting $\bar{T} \equiv T(\bar{\lambda})$, we can then write:

$$\begin{cases} q_t = x(t) = \tilde{x}(c + \bar{\lambda}e^{rt}), & \text{for } t \leq \bar{T} \\ q_t = y(t) = \tilde{x}\left(k(Y^*_{t-\bar{T}}) - \beta^*_{t-\bar{T}}e^{rt}\right) & \text{for } t > \bar{T}.\Box \end{cases}$$