Entry in quota-managed industries: A global game with placement uncertainty

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Abstract

We present a model of firm entry in an industry that is managed with a cap-and-trade quota regulation. Firms are heterogeneous in their individual productivities; each knows its own productivity but is uncertain about where ranks within the firm population. Entry is modeled as a simultaneous move game with incomplete information. Under an industry wide quota, the entry payoff (capital rent) is high if average productivity among the set of all active firms is low. In this case, the quota price is low and the return to vested capital is higher. The opposite holds when the average productivity among the set of active firms is high. We derive a threshold entry strategy which separates active and inactive firms. We show that placement uncertainty in general increases entry relative to a full information benchmark. Additional comparative statics and efficiency implications are provided. We extend our model to consider placement overconfidence, whereby a firm believes it ranks higher on the productivity continuum than is objectively warranted. We show that this form of overconfidence exacerbates the over-entry problem. Our results explain investment/divestment patterns in overcapitalized industries adopting quota regulations, commercial fisheries in particular. The results can also explain excess entry and investment by overconfident entrepreneurs.

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1 Introduction

Tradable quota or cap-and-trade regulation is an increasingly common approach for addressing common pool inefficiencies in industries with negative production externalities (e.g., over-production of environmental bads in polluting industries and excess entry and overharvesting in fisheries). This paper studies entry in a quota-managed industry in a setting where firms differ in terms of their individual productivity levels. In our model firms know their own productivity but are uncertain about the productivity of others, i.e., firms face productivity *placement* uncertainty. Entry is modeled as a simultaneous move game with incomplete information. Atomistic firms first choose whether or not to commit a unit of capital to the quota managed industry. In the second stage firms submit quota demand schedules to a Walrasian auctioneer who announces the equilibrium quota price which equates demand of active firms with fixed (exogenous) supply.

The endogenous quota price divides industry revenues between the capital and variable factors and the fixed quota. Payoff interdependency among firms operates through the equilibrium quota price. Low productivity firms are viable if the set of entrants are also low productivity types, since in this case, the cost of holding quota remains low. In contrast, if the set of entrants has sufficiently high-productivity, quota holding costs are high and low-productivity firms are not viable. Firms do not know the average productivity of competitors, i.e., the decision to enter is modeled as a global game (Carlsson and van Damme, 1993).

Our paper makes several contributions. First, we study the role of heterogeneity, in the form of firm-specific productivity, on resource allocation under private information. A recent and fast growing literature focuses on welfare analysis in such economies highlighting the dual nature of prices - as conveyors of information and as determinants of resource allocation. Moreover, various types of inefficiencies - aggregative or distributional - are traced to externalities arising from this dual role (see e.g. Morris and Shin, 2002, 2005; Angeletos and Pavan, 2007, 2009; Amador and Weil, 2010; Vives, 1993, 1997, 2013). In this literature, an endogenously obtained price function is commonly based on a simplifying assumption, namely, agents are *ex-ante* identical, although they may become different *ex-post* because of the private information they receive. We relax this assumption and explore the role of *ex-ante* heterogeneity in firm entry and exit decisions, quota price formation and the market performance.

Heterogeneity in productivity introduces a level of complexity in the theory of strategic decision making under incomplete information. In our model, agents are differentiated (*ex-ante*) through a marginal cost of production parameter. We assume that population mean cost is unobserved by firms and drawn by Nature from a prior distribution that is common knowledge. These are common features of the global games literature in that players (the set of potential entrants) do not know average cost efficiency of their rivals. We separate from much of the global games literature along three dimensions: (1) permanent heterogeneity among firms, (2) endogenous payoffs, and (3) strategic substitution over the range of equilibrium outcomes.

The Bayesian Nash equilibrium of our market game is a profile of *switching strategies* of the following nature: an individual firm commits capital to the quota-managed industry if its cost parameter is below an endogenously determined threshold, and allocates capital to an alternative use otherwise. The baseline model which assumes that firms form posterior of the industry's costs as if its own cost parameter is at the mean of the distribution, matches the standard framework of a global game with strategic substitutes. There are two innovations however that deserve mention. First, our payoff function is endogenous whereas the literature so far has shown the existence of an equilibrium with an exogenously given payoff function, sometimes by assuming a simple parametric form (see e.g., Karp, Lee and Mason, 2007). Second, the payoff function depends not only on the unobserved fundamental through the endogenous price function, but also on the private signal itself. Propositions 2 and 3 show the existence of a unique pure strategy Bayesian equilibrium for the baseline model. Proposition 1 characterizes a similar equilibrium in switching strategies under full information - that is when mean cost efficiency is common knowledge.

A comparison of the Bayesian equilibrium with the full information equilibrium provides the following main result for the baseline model: the equilibrium threshold marginal cost under incomplete information is higher than its full information counterpart, in an expected sense. The result essentially implies that under incomplete information more cost inefficient firms will enter, relative to the full information case. The result helps partially explain the persistence of inefficient active firms in industries that have adopted quota regulations.

Entry in our model is determined by the firm's belief about its productivity rank or placement within a firm population. Our model provides a natural framework to study overconfidence, more specifically, over-placement bias and industry structure and performance. Individuals including firms managers often over-estimate their ability to complete complex tasks and rank their abilities higher above others and above than is warranted by objective reality. This form of overconfidence can lead to over-investment, and failed business ventures (e.g., Camerer and Lovallo, 1999; Koellinger et al., 2007). In a later section, we extend our model to study the implications of over-placement bias on entry and efficiency in quota managed industries. We show that the equilibrium productivity threshold declines with overplacement bias, i.e., firms whose overconfidence causes them to rank their firm's productivity higher than objectively warranted are more likely to enter the industry. Placement bias reinforces the over-entry problem causing more entry and lower productivity relative to the full information benchmark.

A large literature studies firm dynamics in a variety of settings, e.g., complete and incomplete information. Jovanovic (1982) studies entry (and exit) in a setting where firms are uncertain about their own productivity and learn by doing/producing once entry has occurred. Instead, in our set up firms are aware of their own productivity but uncertain of their competitiveness. The quota regulation, by pricing a negative externality, forces managers to carefully assess their productivity placement before entering the industry.[1]

As in our paper, exit decisions that rely on rivals' characteristics and decisions have also been studied by Ghemawat and Nalebuff (1985, 1990) and Fudenberg and Tirole (1986). Ghemawat and Nalebuff present a model of strategic exit in an exogenously declining industry. The authors derive unique subgame-perfect equilibria in which the larger of the competing firms either exits or reduces its productive capacity because bigger firms suffer greater losses than small firms as demand declines in the industry. In our model the industry size is determined exogenously by an

[1] Our model is inspired by commercial fisheries which, over time, have attracted excess and redundant investment in fishing capital due to the well-known tragedy of the commons problem (Gordon, 1954; Smith, 1969). Managers commonly allocate a binding fishing quota gratis to participating fishermen and allow market forces to remove the excess capital. The process of removing excess capital from the fishery is referred to as fleet rationalization. See Grafton et al., 2000, or Committee to Review Individual Fishing Quotas, 1999, for an review of overcapitalization and quota management programs in U.S. and world fisheries.

aggregate quota regulation. Firms instead compete on productivity for the right to produce the fixed industry output. The return to the vested capital increases with industry size but decreases with rival firm's productivity. Fudenberg and Tirole (1986) introduce incomplete information into a dynamic duopoly competition game. As in the model of our paper, firms know their own costs but must choose when, if ever, to exit based on their beliefs about rival costs that is independent of the firms own realized cost. In our model, firms' cost realizations are independently drawn from a distribution with an unknown mean. Firms use their own cost realizations as a signal to form posterior on the populations' cost distribution and their equilibrium behavior.

The paper is organized as follows. The next section presents the model. Section 3 derives the equilibrium quota price, entry behavior and examines market performance under a benchmark of complete information. Section 4 presents the results under incomplete information. Section 5 compares quota prices, industry structure and market performance under complete *vis à vis* incomplete information. Section 6 examines the effects of placement bias and Section 7 concludes.

2 The model

We assume there is a continuum of firms of unit mass in the population. Each firm is endowed with a unit of physical capital which can be used to produce a valued consumer product. We denote the set of all firms as S.

The productivity of capital varies due to differences in the managerial ability of the owner, i.e., the firm manager. Productivity differences manifest as variation in variable costs of production. We let θ_i denote an inverse productivity parameter for firm i; larger values correspond to higher costs. Let $c(q|\theta_i)$ denote variable cost where q is individual firm production. Variable costs are assumed increasing and strictly convex in q. To simplify the analysis that follows $c(.)$ is assumed to take the following form:

$$c(q|\theta_i) = \theta_i q + \frac{1}{2}\lambda q^2.$$

In the sequel, we will often refer to θ_i as simply the productivity or cost efficiency of firm i.

An industry-wide production quota limits aggregate production to the level, Q in each period; $w_i \geq 0$ will denote an initial quota endowment for firm i.[2]

The opportunity cost of allocating capital to the quota-managed industry is its earning potential in a next highest valued use. We denote this per-period capital cost as $\delta_i \geq 0$ for firm i. We will simplify the analysis, and assume capital costs are common for all firms, i.e., $\delta_i = \delta, \ \forall i \in S$.

We consider a representative production period. Suppose Q has been allocated *gratis* to some subset of S.[3] We focus attention on two key decisions for firm managers, hereafter, just *firms*. The

[2] We imagine an industry where a quota regulation has been adopted to correct overproduction due to a negative externality. Examples include, fisheries and polluting industries. Our analysis will characterize equilibrium industry structure and corresponding costs of production over a range of aggregate quota quantities, Q. Once industry costs are determined, the value of Q that maximizes social welfare is determined by equating associated marginal benefits and marginal costs. We do not specify a benefit function in our model and therefore do not derive the optimal Q in this paper.

[3] The analysis will soon show that the initial allocation of Q is irrelevant for our results as long as a frictionless quota trading market exists.

first is a decision to forego δ and commit the firm's capital to the quota-managed industry. This decision is made by all $i \in S$ at the beginning of the production period. The set of firms who allocate their capital to the quota-managed industry are hereafter referred to as *active* firms. We denote the set of entrants or active firms, $A \subseteq S$.

A second decision involves quota trading in a post entry permit lease market. Quota has no value outside the quota-managed industry, or outside of the fixed production period. We assume all quota holders participate in the quota lease market.

All firms announce a net trade schedule to a market maker. This schedule determines the amount of quota the firm is willing to lease for all possible lease prices. The market maker organizes schedules of active firms and determines the market clearing lease price, r. At this price all active firms are required to transact according to their reported net trade schedule (see Maleug and Yates, 2009 for a similar construction).

We use v_i to denote the net quota leased. If v_i is positive (negative) i is a net buyer (seller) of quota. Because one cannot sell more quota than is held, leasing is constrained by $w_i + v_i \geq 0$.

The cost efficiency θ_i is private information throughout. We will consider two information scenarios for individual firms. In the first, firms know the average cost efficiency in the population. The full information scenario will provide a benchmark for comparing the effects of incomplete information. Under the second scenario, incomplete information, each firm knows its own productivity value but is uncertain about the productivity of others. Firms share a common belief about the productivity distribution in S. Specifically, θ_i is assumed to be uniformly distributed over an interval $[\theta-\varepsilon, \theta+\varepsilon]$, where θ will denote the mean cost parameter for all $i \in S$, and $\varepsilon > 0$. Henceforth we describe θ as the population (over S) mean cost efficiency.

The timing of the game is the following. In the first stage, each firm (player) $i \in S$ allocates its capital either to the quota-managed industry or to its next highest-valued use. In other words, firms decide whether or not to belong to $A \subset S$. The set A is thus endogenous. Simultaneously, all firms in S submit net quota lease schedules to the market maker. The market maker then (re)allocates quota to all the active firms, that is to all $i \in A$. In the second stage, active firms produce output that is no larger than their quota allocation with the goal of maximizing variable profit, the latter also being the rent to the capital that is committed in the quota-managed industry.

We first examine the equilibrium outcome in the quota lease market.

2.1 Leasing behavior and capital rent

Each active firm $i \in A$ chooses v_i to maximize capital rent plus permit trading receipts:

$$\pi_i = \max_{v_i} \left[p(w_i + v_i) - rv_i - \theta_i(w_i + v_i) - \frac{1}{2}\lambda(w_i + v_i)^2 \right] \tag{1}$$

where p denotes the fixed output price.

The Lagrangian for the maximization problem is,

$$L = p(w_i + v_i) - rv_i - \theta_i(w_i + v_i) - \frac{1}{2}\lambda(w_i + v_i)^2 - \mu(-w_i - v_i),$$

where μ is the Lagrange multiplier associated with the constraint on feasible quota sales. As v_i can be of any sign in equilibrium, the Kuhn-Tucker necessary conditions are:

$$p - r - \theta_i - \lambda(w_i + v_i) + \mu = 0, \tag{2a}$$
$$w_i + v_i \geq 0, \quad \mu[w_i + v_i] = 0 \tag{2b}$$
$$\mu \geq 0. \tag{2c}$$

From (2a) and (2b) we derive an expression for net quota demand for firm i:

$$v_i = \begin{cases} \frac{1}{\lambda}[p - r - \theta_i] - w_i & \text{for } i \in A \\ -w_i & \text{for } i \notin A. \end{cases} \tag{3}$$

For an active firm, quota demand is increasing in $p - r$, and decreasing in the cost parameter, θ_i, and in the quota endowment, w_i. Notice that $p - r$ is a post-entry or short run virtual supply price (Neary and Roberts, 1980). A quota-unconstrained firm facing output price $p - r$ would produce $q_i = w_i + v_i$ as determined in 3 in order to maximize its variable profit. The equilibrium quota price r will determine the share of total industry revenue that is paid to allocated capital versus the share that flows to the fixed quota Q, which we next show.

Net lease schedules from all quota holders are combined to determine the market clearing quota lease price. Carrying out this derivation obtains,

$$r = \max\{p - \frac{\int_{i \in A} \theta_i d\theta_i}{A} - \frac{\lambda Q}{A}, 0\}. \tag{4}$$

The equilibrium lease price is zero if the net aggregate demand, $\int_{i \in A} v_i + \int_{i \notin A} v_i$ is strictly negative. When positive, the market clearing quota trading price equals the average marginal profit, as follows. Given the form of the cost function, the marginal cost of firm i producing q_i units is $MC_i = \theta_i + \lambda q_i$. Averaging the marginal cost across active firms yields $\frac{\int_{i \in A} \theta_i d\theta_i}{A} + \frac{\lambda Q}{A}$. Thus equilibrium r, when positive, equals the average marginal profit among the set of active firms.

The equilibrium lease price increases, one-for-one, with the output price. For a given set of active firms A, the lease price is lower when Q is larger and when the marginal costs of production is higher. That is, the higher the average over cost efficiency parameters θ_i's and λ, the lower is the equilibrium quota price. Holding $\int_{i \in A} \theta_i$ fixed, we see that a larger A increases r. This reflects the fact that with fixed Q, a larger A means lower production per firm and higher marginal profit (lower marginal cost) under our decreasing returns technology.

Combining (3) and (4) determines the quantity for each active firm as a function of relative cost efficiency (depending on whether equilibrium permit price is positive or zero):

$$q_i = w_i + v_i = \begin{cases} \frac{1}{\lambda}\left(\bar{\theta}(A) - \theta_i + \lambda \bar{Q}(A)\right) & \text{for } r > 0 \\ \frac{1}{\lambda}(p - \theta_i) & \text{for } r = 0 \end{cases}$$

where $\bar{\theta}(A) = \int_{i \in A} \theta_i d\theta_i / A$ is the average cost efficiency among the set of active firms and $\bar{Q}(A) = Q/A$ is average production. We describe $\bar{\theta}(A)$, henceforth, as the mean cost efficiency amongst the active, as opposed to θ which is the population mean. $\bar{\theta}(A)$, like A is thus endogenously determined. Quantity produced by firm i is thus increasing in the firm's relative cost efficiency, $\bar{\theta}(A) - \theta_i$. q_i also increases with the total available quota relative to the active production capacity, as measured by $\bar{Q}(A)$. Very importantly, the equilibrium does not depend on the initial quota holdings.[4]

Let $\pi(\theta_i|r)$ denote the variable profit or capital rent for firm i, conditional on quota price r. From (3) we are able to rewrite this expression as,

$$\pi(\theta_i|r) = \frac{1}{2\lambda}(p - \theta_i - r)^2 + rw_i \tag{5}$$

Sometimes it may be useful to express the capital rent for an active firm in terms of relative cost efficiency and the set A. When $r > 0$, we have,

$$\pi(\theta_i|A) = \frac{1}{2\lambda}\left(\bar{\theta}(A) - \theta_i + \lambda\bar{Q}(A)\right)^2 + \left(p - \bar{\theta}(A) - \lambda\bar{Q}(A)\right)w_i. \tag{6}$$

The expression above further highlights the dependence of capital rent in the quota-managed industry on the set of active firms, A.

2.2 Capital allocation

The decision to allocate capital to the industry is determined by,

$$\pi(\theta_i|r) = \begin{cases} \frac{1}{2\lambda}(p - \theta_i - r)^2 + rw_i & \text{if } i \in A \\ \delta + rw_i & \text{if } i \notin A \end{cases} \tag{7}$$

The capital allocation decision depends on the value of r that is realized in the second stage. In this paper, the Nash equilibria that we study are also rational expectations equilibria. A firm correctly anticipates the equilibrium quota price (function) and incorporates it into the entry decision. Thus the Nash equilibria are also sub-game perfect.

Further, note that payment to the initial quota endowment w_i is collected regardless of where the firm's capital is employed. That is, in our model, which assumes frictionless quota trades, the capital allocation decision is independent of the individual quota endowments, w_i.[5]

[4] This allows us to identify the factors that affect the decision to be a buyer/seller of quota in the lease market. Rewriting the equation for quota demand, we find

$$v_i = \frac{1}{\lambda}\left(\bar{\theta}(A) - \theta_i\right) + \lambda\bar{Q}(A) - w_i,$$

Then the decision to be a net buyer of quota depends on a comparison of:

1. Relative cost efficiency, $\bar{\theta}(A) - \theta_i$: $\frac{\partial v_i}{\partial(\bar{\theta}(A) - \theta_i)} = \frac{1}{\lambda} > 0$
2. Average production relative to private endowment, $\bar{Q}(A) - w_i$: $\frac{\partial v_i}{\partial(\bar{Q}(A) - w_i)} = 1$

The relative important of cost efficiency and the output share to endowment depends on the slope of the marginal cost curve λ. If $\lambda \in (0, 1)$ (the marginal cost curve is flat) then the relative cost efficiency is the driving force in the decision. If, however, $\lambda > 1$ (the marginal cost curve is steep) then the relative endowment of quota is the driving force.

[5] The conditions under which an initial quota allocation will impact efficiency (in a post-trade equilibrium) in

3 Entry and market equilibrium under full information

In a context of cost heterogeneity, a natural type of strategy to study is the "threshold" or "switching" strategy of a firm. Under such a strategy, a firm commits capital to the quota-managed industry if its cost efficiency parameter θ_i is less than or equal to an *endogenously determined* threshold, denoted θ^*. The firms employs its capital in the outside alternative if $\theta_i > \theta^*$. Let $\sigma(\theta_i)$ denote the probability that a firm with cost parameter θ_i enters the quota-managed industry. We explore the existence of a pure strategy Nash equilibrium such that,

$$\sigma(\theta_i) = \begin{cases} 1, & \text{if } \theta_i \leq \theta^* \\ 0, & \text{if } \theta_i > \theta^* \end{cases}$$

The equilibrium existence is studied under alternative information structures. In this section, we assume full or complete information of the mean cost efficiency parameter θ for all players.

Since θ_i is uniformly distributed over $[\theta - \varepsilon, \theta + \varepsilon]$, under an equilibrium threshold θ^* (if it exists) and for a given θ, the proportion of active firms in the population, $\alpha(\theta, \theta^*)$, have the expression

$$\alpha(\theta, \theta^*) = \begin{cases} 0, & \text{if } \theta^* < \theta - \varepsilon, \text{ or } A = \emptyset \\ \frac{1}{2\varepsilon}\int_{\theta-\varepsilon}^{\theta^*} d\theta_i = \frac{\theta^* - (\theta - \varepsilon)}{2\varepsilon}, & \text{if } \theta - \varepsilon \leq \theta^* \leq \theta + \varepsilon, \text{ or } A \subset S \\ 1, & \text{if } \theta + \varepsilon < \theta^*, \text{ or } A = S \end{cases} \tag{8}$$

The mean cost efficiency amongst active firms has the form,

$$\bar{\theta}(A) = \bar{\theta}(\theta, \theta^*) = \begin{cases} 0, & \text{if } \theta^* < \theta - \varepsilon, \quad \text{as } A = \emptyset \\ \frac{\frac{1}{2\varepsilon}\int_{\theta-\varepsilon}^{\theta^*} \theta_i d\theta_i}{\alpha(\theta, \theta^*)} = \frac{\theta^* + \theta - \varepsilon}{2}, & \text{if } \theta - \varepsilon \leq \theta^* \leq \theta + \varepsilon, \\ \theta, & \text{if } \theta + \varepsilon < \theta^*, \quad \text{as } A = S \end{cases} \tag{9}$$

Then, (4), (8) and (9) may be used to express the equilibrium quota price as a function of the given θ and the endogenous θ^*, as follows.

$$r(\theta, \theta^*) = \begin{cases} \max\{p - \theta - \lambda Q, 0\}, & \text{for } \theta < \theta^* - \varepsilon, \text{ since } A = S \\ \max\{p - \frac{\theta^* + \theta - \varepsilon}{2} - \frac{2\varepsilon\lambda Q}{\theta^* - (\theta - \varepsilon)}, 0\}, & \text{for } \theta^* - \varepsilon \leq \theta \leq \theta^* + \varepsilon, \text{ as } A \subset S \\ 0, & \text{for } \theta^* + \varepsilon < \theta, \text{ since } A = \emptyset \end{cases} \tag{10}$$

For a given θ^*, the equilibrium quota price is decreasing in θ. There are two ways through which θ influences r. First, for a given θ^*, a rise in θ decreases the proportion of active firms, $\alpha(\theta, \theta^*)$. This raises the marginal cost of production under the decreasing returns technology with constant Q. Second, a higher value of θ implies that the mean cost efficiency of the active set, $\bar{\theta}(\theta, \theta^*)$, is also higher.

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cap-and-trade markets has been extensively studied the environmental economics literature. Montgomery (1972) shows that quota market efficiency is independent of the initial quota allocations, when quota trade is frictionless. The initial quota allocation will affect market performance in the presence of transactions costs (Stavins, 1995) or market power (Hahn, 1984; Maleug and Yates, 2009). Our finding the initial quota allocations have no effect on capital investment in quota-managed industries has, to our knowledge, not appeared in earlier literature.

It is also useful to note the relationship between r and θ^* for a given value of θ. So long as $\theta < \theta^* - \varepsilon$, an increase in θ^* has no impact because the set A is unchanged, being equal to S. When $\theta^* - \varepsilon \leq \theta \leq \theta^* + \varepsilon$, for a given θ, a rise in θ^* enlarges the active set bringing in those with higher θ_is. This increases the mean cost efficiency of the active set, $\bar{\theta}(A)$, and exerts a downward pressure on the quota price r. However, under a decreasing returns technology, a larger active set also implies a lower marginal cost as each active firm produces less of the fixed quota and this in turn places an upward pressure on the quota price.

Proposition 1 below shows the existence of a Nash equilibrium in pure threshold strategies, when θ is observable and therefore, firms have complete information about it. Under this scenario, the equilibrium threshold θ^* has a closed form. The form depends on the parametric configuration involving the parameters,λ, Q, δ and ε.

Proposition 1 *1. When $\delta < \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$, there exists a pure strategy Nash equilibrium under complete information of θ. The equilibrium strategy of firm i is given by,*

$$\sigma(\theta_i) = \begin{cases} 1, & \text{if } \ \theta_i \leq \theta^* = p - \sqrt{2\lambda\delta} \\ 0, & \text{if } \ \theta_i > \theta^* \end{cases}$$

2. When $\delta \geq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$, there exists a pure strategy Nash equilibrium under complete information of θ. the equilibrium strategy of firm i given by,

$$\sigma(\theta_i) = \begin{cases} 1, & \text{if } \ \theta_i \leq \theta^*(\theta) \\ & \text{where } \ \theta^*(\theta) = \begin{cases} (\theta - \varepsilon) + \sqrt{2\lambda\delta + 4\varepsilon\lambda Q} - \sqrt{2\lambda\delta} & \text{for } \ \theta \leq \hat{\theta} \\ p - \sqrt{2\lambda\delta} & \text{for } \ \theta > \hat{\theta} \end{cases} \\ 0, & \text{if } \ \theta_i > \theta^*(\theta) \end{cases}$$

where $\hat{\theta} = p + \varepsilon - \sqrt{2\lambda\delta + 4\varepsilon\lambda Q}$.

Under both types of equilibria, an active firm produces a strictly positive quantity.

Proof: See Appendix I.

Proposition 1 provides several useful insights:

Remark 1. When $\delta < \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$, the equilibrium threshold is independent of θ and information of the population mean cost efficiency does not influence the individual entry decision. The reason for this seemingly unintuitive equilibrium feature is as follows. Under this scenario, there is a range of θ values, namely, $\theta \leq p - \varepsilon - \sqrt{2\lambda\delta}$ for which *all* firms find it profitable to employ their capital in the quota-managed industry. This is because the outside capital rent δ is too low. When $A = S$, the equilibrium virtual price $p - r$ equals $\theta + \lambda Q$. Recall that for the highest cost firm in S $\theta_i = \theta + \varepsilon$. Equilibrium profit for the highest cost firm is $\frac{1}{2\lambda}(\theta + \lambda Q - \theta - \varepsilon)^2 = \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$ which is greater than δ. Values of $\theta \geq p - \varepsilon - \sqrt{2\lambda\delta}$ have no effect on individual profitability because (as the Proof shows in detail) the equilibrium permit price attains its minimum value of zero in the full participation region of θ. Thus for firms who continue to be active for $\theta \geq p - \varepsilon - \sqrt{2\lambda\delta}$, the virtual price, $p - r$, is always at its highest level, p. For $\theta \lesseqgtr p - \varepsilon - \sqrt{2\lambda\delta}$, entry decisions are therefore guided purely by the individual θ_is. In sum, the first type of complete information equilibrium thus applies for low capital costs.

REMARK 2. When $\delta > \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$ - that is, when δ is higher than some critical value - for no value of θ is it profitable for *all* firms to be simultaneously active. This scenario is most interesting because under these conditions, a firm chooses a threshold depending on the observed value of θ. θ^* is a linear and increasing function of θ, implying that as the population mean cost parameter θ increases (mean cost efficiency decreases), more high cost types are encouraged to enter.

Note, moreover, that even for very low values of the population mean θ, some firms on the upper end of the distribution always decide to stay out. The reason is the direct decreasing relationship between r and θ for a given θ^* - thus as θ decreases, r increases. Further as θ decreases, $\theta^*(\theta)$ decreases too and casts an increasing influence on r. Although the net effect of a decrease in θ^* on r is thus ambiguous (see earlier discussion) the combined effect of a lower θ and θ^* is such that some firms on the upper end of the distribution always find the equilibrium quota price and their individual cost θ_i to be too high for entry to be profitable.

REMARK 3. Which of the two full information equilibria of Proposition 1 is more likely to occur, is an empirical question. There is one feature of our model, however, which while simplifying the analysis also understates the likelihood of the second type of equilibrium. It is the feature that all firms face a uniform capital cost. A non-uniform δ could make it more likely for some firms not to commit even with low individual θ_i.

REMARK 4. The full information equilibria highlight the strategic role played by the virtual price function $p - r$ in this game. p is assumed exogenously determined. Because of our assumption of a continuum of agents, an individual firm has a negligible impact on the quota price. However, in making his/her entry decision, an individual firm always takes into consideration the effect of others' participation on r and thus on the firm's capital rent. Thus the full information model is akin to a Cournot model with a continuum of agents. Furthermore, the virtual price $p - r$, rather than p per se, plays the important role in this game.

REMARK 5. Note that although under both full information equilibria, the form of the threshold function is identical for a part that applies when $r = 0$, namely $\theta^* = p - \sqrt{2\lambda\delta}$, the threshold values are certain to be different because of different δ values for the same λ and p values. For fixed p and λ, as the value of δ under scenario (2) is certainly greater than the value of δ under scenario (1), $\theta^* = p - \sqrt{2\lambda\delta}$ is certainly lower under scenario (2) than under scenario (1).

4 Entry and market equilibrium under incomplete information with no bias

In the incomplete information version of the game, at stage one, Nature chooses the population mean cost efficiency, θ, which is unobserved by the individual firms. Firm i' knows its own cost parameter θ_i. The uniform distribution of the θ_is around the unobserved mean θ (with noise, ε) is common knowledge. Consistent with the standard global games framework, we assume θ to have a *prior* (improper) uniform distribution over the real space $\mathbf{R}$, from which Nature picks a value. As is well known, however, the prior distribution of θ plays no part in the equilibrium, eventually.

Each firm forms a *posterior* on the distribution of the unknown population mean cost efficiency, θ, given the private signal it has received about its own cost efficiency, θ_i. In this section, we assume that every firm believes itself to be the population average. In other words, a firm with cost efficiency θ_i, believes that θ is posteriori uniformly distributed over $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. In particular, firm i believes that $E(\theta) = \theta_i$.

Based on its private signal, θ_i, firm i believes that the signal received by any other firm j is symmetrically (but not uniformly) distributed over $[\theta_i - 2\varepsilon, \theta_i + 2\varepsilon]$. Thus, the interval $[\theta_i - 2\varepsilon, \theta_i + 2\varepsilon]$ provides, a posteriori (according to firm i), the range of possible cost parameters of the other firms. Note however, the although the posterior distribution of θ_j is symmetric on $[\theta_i - 2\varepsilon, \theta_i + 2\varepsilon]$, it is not uniform over this range. The posterior distribution of the θ_j plays an important role in defining the expected variable profit function of firm i under incomplete information.

As we proceed to discuss the equilibrium threshold strategy under incomplete information, the issue of *post-entry* capital idleness becomes important. When the population mean cost efficiency θ is unknown, the capital allocation decision must be based on *expected* capital rent. Under uncertainty, a firm may potentially find its value of θ_i to be too high relative to the *ex-post* virtual price $p - r(\theta, \theta^*)$, so that it is optimal to keep the capital (it has already committed) idle and not produce at all. We need to rule out this possibility through a parametric restriction under which post-entry production and thus capital rent is strictly positive for firms that have chosen to be active at the first stage. A sufficient condition that guarantees ex-post production and variable profits are strictly positive is

Assumption 1 (No-idleness condition) $\lambda Q > \varepsilon$.

Appendix 10.1 shows why this condition is sufficient. The restriction in particular implies that an equilibrium without ex-post idleness of capital may not exist for any size of the noise parameter, ε.

4.1 Equilibrium threshold strategy

Assume all firms follow a threshold strategy with threshold value θ^*. The expected capital rent for a θ_i-type firm depends on the quota price and A, and takes one of three forms:

$$\begin{array}{lll} a. & \big(r(\theta,\theta^*) > 0,\ A = S\big): & \pi(\theta_i) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \frac{1}{2\lambda}(\theta + \lambda Q - \theta_i)^2 d\theta \\ b. & \big(r(\theta,\theta^*) > 0,\ A \subset S\big): & \pi(\theta_i) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \frac{1}{2\lambda}\left(\frac{\theta^*+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{\theta^*-(\theta-\varepsilon)} - \theta_i\right)^2 d\theta \\ c. & \big(r(\theta,\theta^*) = 0\big): & \pi(\theta_i) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \frac{1}{2\lambda}(p - \theta_i)^2 d\theta \end{array} \quad (11)$$

We seek a symmetric pure strategy Bayesian Nash equilibrium under which all firms adopt the identical threshold strategy with threshold value θ^*. Such a θ^* has the following property. If a firm receives a private signal $\theta_i = \theta^*$, then its expected capital rent is equal to the return from the alternative, δ. Thus to determine θ^*, we first derive the expected capital rent for a $\theta_i = \theta^*$-type firm.

For the $\theta_i = \theta^*$ firm, the posterior distribution for unknown θ is uniform on $[\theta^* - \varepsilon, \theta^* + \varepsilon]$. Moreover, the probability that all firms are active is zero, for the following reason. For such a firm, the lowest value that θ can attain is $\theta^* - \varepsilon$. In this case, the firm is also the highest cost firm in S and $A = S$. Since, we are dealing with a continuous distribution, the probability of such an event is zero. For all other possible values of θ, some firms will have cost parameters higher than θ^* and hence will not be active. Therefore, for a firm with $\theta_i = \theta^*$, depending on whether $r(\theta, \theta^*)$ is strictly positive or zero, the expected capital rent follows form $b.$ or $c.$ in equation (11) or some combination of the two.

Analogous to the full information scenario, the characteristics of the incomplete information equilibrium threshold θ^* depend on the parametric configuration involving the parameters, δ, λ, Q and ε. Depending on this configuration, we have either an equilibrium with (1) $r(\theta, \theta^*) > 0$ for some (or all) $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ or (2) with $r(\theta, \theta^*) = 0$ for all $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$.

A necessary condition for an equilibrium with $r(\theta, \theta^*) > 0$ for some (or all) $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ is that, $r(\theta^* - \varepsilon, \theta^*) > 0$. Substituting $\theta = \theta^* - \varepsilon$ into the (10), the necessary condition is,

$$r(\theta^* - \varepsilon, \theta^*) = p - (\theta^* - \varepsilon) - \lambda Q > 0, \Longrightarrow p - \theta^* > \lambda Q - \varepsilon \tag{12}$$

Similarly, a necessary condition for an equilibrium with $r(\theta, \theta^*) = 0$ for all $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ is that, $r(\theta^* - \varepsilon, \theta^*) = 0$. From the equilibrium quota price function (10), the necessary condition reduces to,

$$p - (\theta^* - \varepsilon) - \lambda Q \leq 0, \Longrightarrow p - \theta^* \leq \lambda Q - \varepsilon \tag{13}$$

Conditions (12) and (13) thus divide up the parameter space into two zones. When $r(\theta^* - \varepsilon, \theta^*) = 0$, the expected profit of the firm with realization $\theta_i = \theta^*$ is given by,

$$\frac{1}{2\varepsilon} \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} \frac{1}{2\lambda} (p - \theta^*)^2 d\theta = \frac{1}{2\lambda} (p - \theta^*)^2 = \delta$$

Then from condition (13), $\delta = \frac{1}{2\lambda}(p - \theta^*)^2 \leq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$.

Thus the necessary parametric condition for an equilibrium with $r(\theta, \theta^*) = 0$ for all $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ is that $\delta \leq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$. Similarly, $\delta > \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$ is a necessary condition for the existence of an equilibrium θ^* with $r(\theta^* - \varepsilon, \theta^*) > 0$ for some $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$.

The incomplete information equilibrium is therefore differently characterized for two different regions in which δ may lie: (1) $\delta \in \left(\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}\right]$, and (2) $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2\right]$.

4.2 $\delta \in \left(\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}\right]$

For any given θ^*, the quota price function, $r(\theta, \theta^*) = p - \frac{\theta^* + \theta - \varepsilon}{2} - \frac{2\varepsilon\lambda Q}{\theta^* - (\theta - \varepsilon)}$ attains zero at a value of $\theta < \theta^* + \varepsilon$, since p is finite. Given θ^*, the roots of $p - \frac{\theta^* + \theta - \varepsilon}{2} - \frac{2\varepsilon\lambda Q}{\theta^* - (\theta - \varepsilon)} = 0$ are given by $\hat{\theta}(\theta^*) = (p + \varepsilon) \pm \sqrt{(p - \theta^*)^2 + 4\varepsilon\lambda Q}$. The restriction $\hat{\theta}(\theta^*) < \theta^* + \varepsilon$ implies that only the root $\hat{\theta}(\theta^*) = (p + \varepsilon) - \sqrt{(p - \theta^*)^2 + 4\varepsilon\lambda Q}$ need be considered. Moreover by condition (12), $\hat{\theta}(\theta^*) > \theta^* - \varepsilon$, implying $\theta^* - \varepsilon < \hat{\theta}(\theta^*) < \theta^* + \varepsilon$, for a given θ^*.

Thus the expected profit of a firm with $\theta_i = \theta^*$ is given by,

$$\frac{1}{2\varepsilon}\int_{\theta^*-\varepsilon}^{\hat{\theta}(\theta^*)}\frac{1}{2\lambda}(\frac{\theta^*+\theta-\varepsilon}{2}+\frac{2\varepsilon\lambda Q}{\theta^*-(\theta-\varepsilon)}-\theta^*)^2d\theta+\frac{1}{2\varepsilon}\int_{\hat{\theta}(\theta^*)}^{\theta^*+\varepsilon}\frac{1}{2\lambda}(p-\theta^*)^2d\theta)$$

The following Proposition characterizes the incomplete information equilibrium for this region.

Proposition 2 *A unique pure strategy Bayesian Nash equilibrium in switching strategies exists for every* $\delta \in \left(\frac{1}{2\lambda}(\lambda Q-\varepsilon)^2, \frac{(\lambda Q)^2}{2\lambda}+\frac{\varepsilon^2}{6\lambda}\right]$. *The equilibrium strategy has the form*

$$\sigma(\theta_i)=\begin{cases} 1, & \textit{if} \;\; \theta_i \leq \theta^*(\delta) \\ 0, & \textit{if} \;\; \theta_i > \theta^*(\delta) \end{cases}$$

where $\theta^*(\delta)$ *solves the following equations:*

$$\frac{1}{4\lambda\varepsilon}[\int_{\theta^*-\varepsilon}^{\hat{\theta}(\theta^*)}(\frac{\theta^*+\theta-\varepsilon}{2}+\frac{2\varepsilon\lambda Q}{\theta^*-(\theta-\varepsilon)}-\theta^*)^2d\theta+\int_{\hat{\theta}(\theta^*)}^{\theta^*+\varepsilon}(p-\theta^*)^2d\theta)]=\delta \tag{14}$$

$$p-(\theta^*-\varepsilon)-\lambda Q>0 \tag{15}$$

Proof: See Appendix II.

4.3 $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q-\varepsilon)^2\right]$

The following Proposition characterizes the incomplete information equilibrium for this region.

Proposition 3 *A unique pure strategy Bayesian Nash equilibrium in switching strategies exists for every* $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q-\varepsilon)^2\right]$. *The equilibrium strategy has the form*

$$\sigma(\theta_i)=\begin{cases} 1, & \textit{if} \;\; \theta_i \leq \theta^*(\delta) \\ 0, & \textit{if} \;\; \theta_i > \theta^*(\delta) \end{cases}$$

where $\theta^*(\delta)$ *solves the following equations:*

$$\frac{1}{4\lambda\varepsilon}[\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon}(p-\theta^*)^2=\delta \tag{16}$$

$$p-(\theta^*-\varepsilon)-\lambda Q\leq 0 \tag{17}$$

Proof: See Appendix II.

4.4 Incomplete information equilibrium with no bias

Remark 6: Propositions 1, 2, and 3 show that uncertainty regarding firms' efficiency rank matters for capital investment, only when $\delta \geq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$. Under the alternative scenario, $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2\right)$, the equilibrium threshold θ^* is identical under full and incomplete information, implying that for a given θ, the set of active firms is identical under both information structures. This is because, the equilibrium quota price $r(\theta, \theta^*)$ - through which the θ influences the determination of θ^* - is zero, whether θ is observed or unobserved.

Thus placement uncertainty does not always matter for entry decision. It matters when the opportunity cost of capital or return from its best alternative use, δ, is higher than a critical level. A high enough δ provides a justification for the less cost efficient firm to sell off their quotas to more cost efficient firms.

Remark 7: The present section characterizes the incomplete information equilibria under the an assumed belief on the part of each firm with regards to its rank in the population distribution, namely each firm i regards itself as the population mean. Many studies suggest however that this may not be the most prevalent attitude amongst potential entrants in the face of placement uncertainty. Section 6 extends the characterization of the equilibria to the situation when firms are over-confident about their efficiencies and regard themselves as above average.

5 Active mass: incomplete vis-à-vis complete information

In this section we discuss the effect of placement uncertainty on the mass of active firms in an industry. The mass of entrants serves as a surrogate for total amount of capital invested in the industry.

Conventional wisdom would lead us to expect too many cost inefficient entrants into the industry if potential entrants know their own cost efficiency but not the population average efficiency, compared to a situation when they know both. In terms of our model, we should expect the mass of active firms A to be larger if θ is unobserved compared to a situation when it is observed. In this section, we investigate whether this conventional wisdom is true.

The mass of active firms, A, is given by:

$$\alpha(\theta, \theta^*) = \begin{cases} 0, & \text{if } \theta^* < \theta - \varepsilon, \text{ none active} \\ \frac{1}{2\varepsilon}\int_{\theta-\varepsilon}^{\theta^*} d\theta_i = \frac{\theta^* - (\theta - \varepsilon)}{2\varepsilon}, & \text{if } \theta - \varepsilon \leq \theta^* \leq \theta + \varepsilon, \text{ some active} \\ 1, & \text{if } \theta + \varepsilon < \theta^*, \text{ all active} \end{cases} \quad (18)$$

In what follows, we use $\left\{\theta^{*f}, \alpha^f\right\}$ and $\left\{\theta^{*^i}, \alpha^i\right\}$ to differentiate the thresholds and active masses of firms under full and incomplete information, respectively. Furthermore, note that these thresholds and active masses are functions of several parameters, in particular, the parameter of special interest, δ.

Remark 6 points out that θ^{*f} and θ^{*i} are identical when $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2\right]$ and differ for, $\delta \in \left[\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2, \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda}\right]$. The issue of possibly excessive entry or overcapitalization under incomplete information is thus meaningful only when δ lies in the latter region.

From equation (18), the realized mass of firms depends (1) on entry threshold θ^* and (2) the realization of the population mean, θ. Under incomplete information, the entry threshold is independent of θ. Under full information, it is a linear and increasing function of θ. Thus, θ^{*f} may be greater, less or equal to θ^{*i}, depending on the realization of θ. The active mass under the two information structures, can therefore be compared only in an *expected* sense. In other words, the comparables for our study are the expected magnitudes, $E(\alpha^f)$ and $E(\alpha^i)$ over all possible values of θ. We thus need to address the issue of the actual distribution of θ first.

We assumed in the introduction that θ is a priori (improper) uniformly distributed on $\mathbf{R}$ and that this is common knowledge. As we have seen, the equilibrium threshold, θ^{*i}, however does not depend on the nature of this prior distribution of θ. The assumption is more of a modelling convenience that is standard in the literature and is dispensable. The assumption that is critical for us, specifically, critical for the determination of θ^{*i} is that the uniform distribution of the θ_i around the unknown population mean, θ, is common knowledge.

For a meaningful comparative study of the active masses, it is important to assume that θ has a bounded distribution, with θ^l as the lower bound and θ^h, the upper bound. More specifically, as we explain towards the end of section 5.2 below, the comparative static result (Proposition 4) becomes trivial and therefore less interesting without a lower bound on the actual θ. Furthermore, from a purely empirical point of view, bounds are more realistic.

It is nevertheless important to propose a way to reconcile the two different views about the actual distribution of θ - the view laid out in section 4 that $\theta \in \mathbf{R}$ and the present one of a bounded θ. To this end, we assume that the actual distribution of θ, $\theta \in [\theta^l, \theta^h]$ is known only to the modeler. The agents believe, without ramification for equilibrium behavior, that θ is unbounded on $\mathbf{R}$.

The next step is to specify likely values for the upper and lower bounds, θ^h and θ^l. We show below that the active mass, α equals zero for $\theta \geq p - \sqrt{2\delta\lambda} + \varepsilon$, with or without complete information, implying that both α^{*i} and α^{*f} are both zero and hence their difference is zero. We may therefore, without loss of generality, assume that the upper bound for θ is $\theta^h = p + \varepsilon - \sqrt{2\delta\lambda}$.

By contrast, there is no unique natural choice for θ^l. We need to lay out an admissible *range* of values for θ^l. As θ is the intercept of the marginal cost of a firm, a value of $\theta^l = \varepsilon$, implies that the resulting distribution of θ_i's is given by, $\theta_i \sim U[0, 2\varepsilon]$. In other words, when $\theta^l = \varepsilon$, the lowest cost firm in the distribution has a zero intercept. We may assume, without loss of generality, that firms do not have negative marginal costs and therefore, the lowest admissible value of θ^l is as low as ε.

On the other hand, the highest admissible value for θ^l is $\theta^{*i} - \varepsilon$, for a given θ^*, as the incomplete information equilibrium is defined for admissible values of $\theta \in [\theta^{*i} - \varepsilon, \theta^{*i} + \varepsilon]$.

Our comparative study is therefore based on the following distribution for θ: $\theta \sim U[\theta^l, \theta^h]$, where $\theta^h = p + \varepsilon - \sqrt{2\delta\varepsilon}$ and $\theta^l \in [\varepsilon, \theta^{*i} - \varepsilon]$ for a given θ^{*i}. Thus the assumed distribution of θ has a support that may be as wide as $[\varepsilon, p + \varepsilon - \sqrt{2\delta\varepsilon}]$ or as narrow as $[\theta^{*i} - \varepsilon, p + \varepsilon - \sqrt{2\delta\varepsilon}]$ for a given θ^{*i}.

5.1 Difference in active mass

Fix a δ and a θ^l. Following Proposition 1, the active mass under full information has the form,

$$\alpha^f(\delta) = \begin{cases} \frac{\sqrt{2\delta\lambda+4\varepsilon\lambda Q}-\sqrt{2\delta\lambda}}{2\varepsilon} < 1, \text{ for } \theta \leq \hat{\theta} \\ \frac{(p-\sqrt{2\delta\lambda})-(\theta+\varepsilon)}{2\varepsilon}, \quad \theta \in \left[\hat{\theta}, p+\varepsilon-\sqrt{2\delta\lambda}\right] \\ 0, \quad \theta > p-\sqrt{2\delta\lambda}+\varepsilon. \end{cases} \tag{19}$$

where, it was shown previously, $\hat{\theta}(\delta) = p+\varepsilon-\sqrt{2\lambda\delta+4\varepsilon\lambda Q}$.

As $\theta^{*i}(\delta)$, does not have a closed form, an analogous expression for $\alpha^i(\delta)$ may not be found. It is nevertheless possible to formalize the difference between the two active masses, $(\alpha^i(\delta)-\alpha^f(\delta))$, analytically.

The function, $(\alpha^i(\delta)-\alpha^f(\delta))$ turns out to be discontinuous but linear in θ and consists of different segments. The form differs slightly depending on whether (1) $\hat{\theta}(\delta) \leq \theta^{*i}(\delta)+\epsilon$ or (2) $\theta^{*i}(\delta)+\epsilon \leq \hat{\theta}(\delta)$ is true. Depending on the values of the parameters, both inequalities are possible. However, as we explain below, the main result of the paper (Proposition 4) does not depend upon which specific inequality holds.

Details of both forms are derived and discussed in Appendix III. For the purposes of presenting and exposition of our main result, the function is illustrated here, for a specific value of δ and under the assumption that inequality (1) holds.

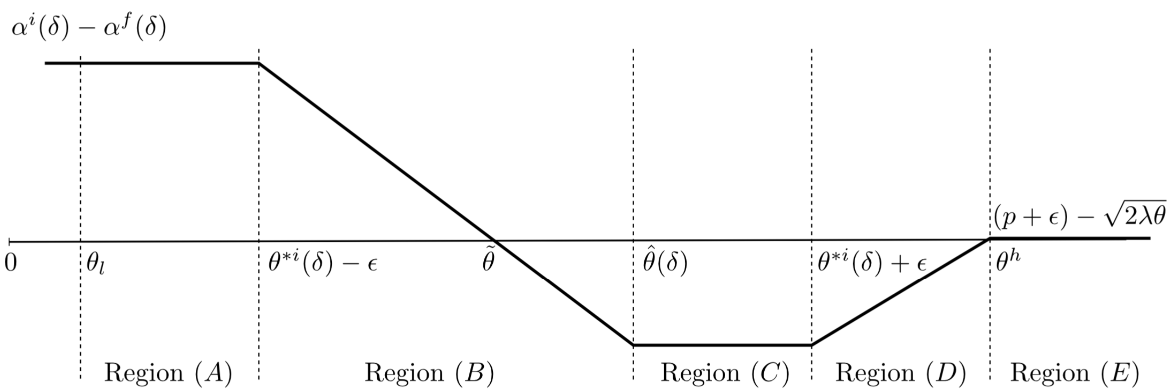

Figure 1: $(\alpha^i(\delta)-\alpha^f(\delta))$: Case 1

The following features of the function, $\alpha^i-\alpha^f$, are worth noting. First, the function takes on positive as well as negative values depending on θ. Thus, depending on the value of θ, the active mass under incomplete information could be higher than, equal to or lower than the active mass under complete information.

Our interest is in the net area under the curve which represents the *expected* difference in the active masses. However, there is no obvious reason why the positive area under the curve must be greater than the negative area. The value of the function over the different zones (A) - (D), and the position of the cardinal points, $\theta^{*i}-\varepsilon$, $\hat{\theta}$, $\theta^{*i}+\varepsilon$, $\theta^h = p-\sqrt{2\delta\lambda}+\varepsilon$ and the point $\tilde{\theta}$ at which the function crosses the zero line, all depend in a complicated way on the parameters of the model, in particular, on the parameter of interest, δ. Thus, there is no obvious reason for the conventional wisdom about excessive entry under incomplete information to be true.

Second, assuming no upper bound on θ or having θ unbounded above, does not materially affect the net area under this function because the function has a value of zero for all $\theta > \theta^h = p-\sqrt{2\delta\lambda}+\varepsilon$. On the other hand, assuming no lower bound on θ increases the positive area under the function indefinitely. Thus conventional wisdom holds trivially true in the special case when the lower bound θ^l is sufficiently low or does not exist. Such cases, moreover, may also be less interesting from a realistic point of view. Thus the results of this section draw their main appeal from the bounds placed on θ, in particular, the lower bound on θ.

The net area under the function, $\alpha^i-\alpha^f$, represents the expected difference in active masses. It is straightforward to show that for a given θ^l and δ, this area is given by,

$$E(\alpha^i-\alpha^f) = (\theta^{*i}(\delta)-\theta^l)+\lambda Q-(p+\varepsilon-\theta^l)\left(\frac{\sqrt{2\delta\lambda+4\varepsilon\lambda Q}-\sqrt{2\delta\lambda}}{2\varepsilon}\right) \tag{20}$$

Appendix III provides details of all calculations.

Proposition 4 *Denote* $\frac{1}{2\lambda}(\lambda Q-\varepsilon)^2=\delta^l$. *For every* $\theta^l \in \left[\varepsilon, \theta^{*i}(\delta^l)-\varepsilon\right]$, *there exists an interval* $(\delta^l, \hat{\delta}(\theta^l))$, *such that if* $\delta \in (\delta^l, \hat{\delta}(\theta^l))$, $E(\alpha^i-\alpha^f)>0$.

Proof: The right side of expression (20) is 0 for all $\theta^l \in [\varepsilon, \theta^{*i}-\varepsilon]$ if $\delta=\delta^l$ (See Appendix III for details). The derivative of expression (20) at $\delta=\delta^l$ is given by (see Appendix III for details),

$$\frac{\partial E(\alpha^i-\alpha^f)}{\partial\delta} = \frac{\lambda}{\lambda Q-\varepsilon}\left(\frac{p+\varepsilon-\theta^l}{\lambda Q+\varepsilon}-1\right) \tag{21}$$

The right side of equation (21) is strictly positive iff $\theta^l < p-\lambda Q$ and equal to zero if $\theta^l = p-\lambda Q$. For $\delta=\delta^l$ and $\theta^l=\theta^{*i}(\delta^l)-\varepsilon$, it can be shown that $\theta^l=\theta^{*i}(\delta^l)-\varepsilon = p-\lambda Q$. Hence the derivative is zero for $\theta^l=\theta^{*i}(\delta^l)-\varepsilon$. As, $\frac{\partial^2 E(\alpha^i-\alpha^f)}{\partial\theta^l\partial\delta}<0$, for $\theta^l<\theta^{*i}(\delta^l)-\varepsilon = p-\lambda Q$, the derivative is strictly positive.

Hence for every $\theta^l \in \left[\varepsilon, \theta^{*i}(\delta^l)-\varepsilon\right]$, there exist values of $\delta>\delta^l$ but sufficiently close to it for which $E(\alpha^i-\alpha^f)>0$. Hence, for every $\theta^l \in \left[\varepsilon, \theta^{*i}(\delta^l)-\varepsilon\right]$, there exists an interval $(\delta^l, \hat{\delta}(\theta^l))$, such that if $\delta \in (\delta^l, \hat{\delta}(\theta^l))$, $E(\alpha^i-\alpha^f)>0$. Note, that the interval depends on θ^l. Further, as $\theta^{*i}(\delta)$ is decreasing in δ, $\theta^{*i}(\delta) - \varepsilon < \theta^{*i}(\delta^l) - \varepsilon$ for $\delta > \delta^l$ and the range of admissible values of θ^l for any such interval is a strict subset of $\left[\varepsilon, \theta^{*i}(\delta^l) - \varepsilon\right]$. This completes our proof. Δ.

Remark 8: Proposition 4 shows that over entry of firms or over investment of capital is possible under incomplete information about the average cost efficiency, for values of δ within a range, namely $\delta \in (\delta^l, \hat{\delta}(\theta^l))$. If the return on the alternative use of the capital, δ, is very low - that is less than a critical level, δ^l - information or the lack of it, about the mean cost efficiency, does not matter for entry decisions. A firm's entry decision is based on its own cost efficiency, θ_i, only as the expected permit price - the instrument through which the set of active firms affect an individual firm's entry decision - is zero.

6 Placement Bias

In this section, we study how the incomplete information equilibrium changes when we relax the assumption that an agent unsure of his relative rank in a population, assumes that he is the average. Instead, we assume that all agents are over-confident about their abilities and assume that they have above average skills. In other words, an agent observing his cost parameter θ_i, believes that the unobserved mean cost parameter, θ lies in the range, $[\theta_i + \beta - \varepsilon, \theta_i + \beta + \varepsilon]$ where $\beta > 0$. Specifically, the agent believes that $\theta \sim U[\theta_i + \beta - \varepsilon, \theta_i + \beta + \varepsilon]$, implying $E(\theta) = \theta_i + \beta > \theta_i$. Recall that under the previous scenario, in the absence of over-confidence, an agent believes that $E(\theta) = \theta_i$.

The parameter β provides a measure of the agent's over-confidence and is henceforth, variously described as *placement bias*, *confidence bias* or *bias* for short. We assume that β is uniform across all agents. We assume further that although an agent is unaware that his belief about himself is the result of a bias (he believes that he is truly superior) he knows that all other agents have a bias. In other words, we assume that β is common knowledge.

Recall that a fundamental assumption of the model is that the distribution of the individual θ_i's around the unobserved θ is common knowledge - that is, $\theta_i \sim U[\theta - \varepsilon, \theta + \varepsilon]$ is common knowledge. The two assumptions of common knowledge are consistent with each other only if $\beta \leq 2\varepsilon$, for the following reason.

If $\theta_i \sim U[\theta - \varepsilon, \theta + \varepsilon]$ is common knowledge, all agents know that their individual θ_i cannot be less than $\theta - \varepsilon$. Since for $\beta > 0$, the minimum $\theta = \theta_i + \beta - \varepsilon$, the former implies that all agents know that θ_i cannot be less than $\theta_i + \beta - 2\varepsilon$ or $\beta - 2\varepsilon$ cannot be strictly positive. In other words, $\beta \leq 2\varepsilon$. Thus any arbitrarily large measure of over-confidence is not consistent with common knowledge of the distribution of the individual θ_is.

Further note that when $\beta = \varepsilon$, an agent with cost parameter θ_i, believes that $\theta \sim U[\theta_i, \theta_i + 2\varepsilon]$, implying $E(\theta) = \theta_i + \varepsilon$ or $\theta_i = E(\theta - \varepsilon)$. In other words, he believes that he has the lowest cost parameter in a population that is uniformly distributed around an unknown mean. Whereas, if $\beta > \varepsilon$, $E(\theta - \varepsilon) = \theta_i + \beta - \varepsilon > \theta_i$ - the agent believes himself to be an outlier. Moreover, as θ_i is arbitrary, all agents believe themselves to be outliers. This is a very special and we think, an unlikely scenario. Therefore, although not crucial for the existence results, we shall implicitly assume that $\beta \leq \varepsilon$.

As before, we explore the existence of a threshold θ_i, denoted μ^* (to differentiate from the no-bias equilibrium θ^*), such that firms commit capital if $\theta_i \leq \mu^*$ and do not commit, otherwise. Note that with over-confidence bias $\beta > 0$, the expressions for the proportion of active firms, the mean cost efficiency of active firms and the equilibrium quota price have the same form as before - namely, expressions (8), (9) and (10) respectively - except that, θ^* is replaced by μ^*. Upon learning its cost parameter $\theta_i = \mu^*$, a firm believes that $\theta \sim U[\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$. Equilibrium μ^* is thus characterized by,

$$\frac{1}{2\varepsilon}\int_{\mu^*+\beta-\varepsilon}^{\mu^*+\beta+\varepsilon} \pi(\theta, \mu^*)d\theta = \delta$$

Notably, equilibrium μ^* depends in general not only on δ but on the parameter β as well, that is $\mu^* = \mu^*(\delta, \beta)$.

As in the case of $\beta = 0$, we explore two types of equilibrium - (1) an equilibrium characterized by positive permit price for some values of $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$ and (2) an equilibrium characterized by zero permit price for all values of $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$.

We begin by noting that the permit price function $r(\theta, \mu^*)$ has the same functional form irrespective of the value of β. In particular, $r(\theta, \mu^*)$ is influenced by β only through μ^*.

Hence $r(\theta, \mu^*) = 0$ for all $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$, iff

$$r(\mu^* + \beta - \varepsilon, \mu^*) = 0$$

which on substitution is equivalent to the condition

$$p - \frac{2\mu^* + \beta - 2\varepsilon}{2} - \frac{2\varepsilon\lambda Q}{2\varepsilon - \beta} \leq 0$$

$$\implies p - \mu^* \leq \frac{2\varepsilon\lambda Q}{2\varepsilon - \beta} - \frac{2\varepsilon - \beta}{2}$$

When $r(\theta, \mu^*) = 0$ for all $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$, the equilibrium μ^* is given by the solution of

$$\frac{1}{2\varepsilon}\int_{\mu^*+\beta-\varepsilon}^{\mu^*+\beta+\varepsilon} \frac{1}{2\lambda}(p - \mu^*)^2 d\theta = \delta$$

$$\implies \mu^* = p - \sqrt{2\lambda\delta}$$

Substituting the expression for μ^* in the previous expression, we obtain the following necessary condition on the parameters that must be satisfied for the type (2) equilibrium.

$$\delta \leq \frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon - \beta} - \frac{2\varepsilon - \beta}{2})^2$$

Thus as in the case of $\beta = 0$, we identify two regions in which δ may lie, each region supporting a different type of equilibrium.

6.1 $\delta \in (\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}]$

Assuming that the interval $[\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}]$ can be shown to be non-empty (see Step I of Proof in Appendix), the equilibrium permit price, $r(\theta, \mu^*)$, is strictly positive for some values of $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$, if δ lies in this region.

The following Proposition characterizes the incomplete information equilibrium with bias for values of δ in this region:

Proposition 5 *(1). A pure strategy Bayesian Nash equilibrium in switching strategies exist for values of $\delta \in (\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}]$. The equilibrium threshold μ^* such that a firm commits if $\theta_i \leq \mu^*$ and does not commit otherwise, is the unique solution of the following equations:*

$$\frac{1}{4\lambda\varepsilon}[\int_{\mu^*+\beta-\varepsilon}^{\hat{\theta}(\mu^*)}(\frac{\mu^*+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{\mu^*-(\theta-\varepsilon)} - \mu^*)^2 d\theta + \int_{\hat{\theta}(\mu^*)}^{\mu^*+\beta+\varepsilon}(p-\mu^*)^2 d\theta)] = \delta \tag{22}$$

$$p - \mu^* > (\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2}) \tag{23}$$

where $\hat{\theta}(\mu^*) = (p+\varepsilon) - \sqrt{(p-\mu^*)^2 + 4\varepsilon\lambda Q}$. In addition, $\mu^*(\delta, \beta) > \theta^{*i}$ for $\beta > 0$ (2). For $\delta \leq \frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2$, the equilibrium μ^*, such that a firm commits if $\theta_i \leq \mu^*$ and does not commit otherwise, is given by Equilibrium $\mu^* = p - \sqrt{2\lambda\delta}$.

Proof: See Appendix IV.

Remark 9: The equilibrium has the feature that the permit price is zero for all values of $\theta \in U[\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$. Moreover, the equilibrium μ^* does *not* depend on β. The firms choose the same threshold, whether or not they have biased beliefs about themselves.

Remark 10: The proofs of both Propositions are very similar to the corresponding no-bias cases, with minor differences in expression. The major difference with the no-bias case lies in the fact that existence is guaranteed for a different range for δ. Specifically, $\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2 < \delta < \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda}$, where $\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2 > \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$ and $\frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda} > \frac{(\lambda Q)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}$, the previous upper bound for δ. Thus, both upper and lower bounds for the range of δ values are higher than in the previous case.

Remark 11: $\mu^*(\delta, \beta)$ is determined by the solution of (22) in the above proposition. The expression within brackets in the first term is increasing and convex in θ. Hence, as the limits of the integrals are increasing in β, $\pi(k, \beta)$ is increasing in β. Hence, the solution is increasing in β.

7 Conclusion

We present a model of firm entry in an industry that is managed with a cap-and-trade quota regulation. Firms know their own productivity but are uncertain about their ranks within the population. Entry is modeled as a simultaneous move game with incomplete information. We derive a threshold entry strategy which separates active and inactive firms. We show that placement uncertainty in general increases entry relative to a full information benchmark. Additional comparative statics and efficiency implications are provided. We extend our model to consider placement overconfidence, whereby a firm believes it ranks higher on the productivity continuum than is objectively warranted. We show that this form of overconfidence exacerbates the over-entry problem.

We have considered a one shot game of entry under placement uncertainty. Firms in our model observe the quota price only after the entry/exit decision is made. In fact, quota prices convey information about population productivities. If the model is extended to multiple periods, full industry adjustment may take several periods if, for example, firms choose to delay entry in attempts to resolve productivity placement bias. Costly reversibility of capital investments combined with learning as in, for example Jovanovic (1982), could provide additional results for industry adjustment in quota-regulated industries. This is left for future work.

8 References

Angeletos G. and I. Werning, 2006, Crises and Prices: Information Aggregation, Multiplicity, and Volatility, American Economic Review, 96, 1720-36.

Amador and Weil, 2010, Learning from prices: Public communication and welfare, Journal of Political Economy, 118(5), 866-907

Angeletos and Pavan, 2007, Efficient use of information and social value of information, Econometrica, 75(4), 1103-1142

Angeletos and Pavan, 2009, Policy with Dispersed information, Journal of European Economic Association, 7(1), 11-60

Carlsson H. and E. van Damme (1993), Global Games and Equilibrium Selection, Econometrica 61, 989-1018.

Committee to Review Individual Fishing Quotas, 1999, Sharing the Fish: Toward a National Policy on Individual Fishing Quotas, National Academy Press, Washington, D.C.

Fudenberg, D. and J. Tirole, 1986, A Theory of Exit in Duopoly, Econometrica, 54, 943-60.

Ghemawat, P. and B. Nalebuff, 1985, Exit, Rand Journal of Economics, XVI (Spring), 184-94.

Ghemawat, P. and B. Nalebuff, 1990, The Devolution of Declining Industries, Quarterly Journal of Economics, 105, 167-186.

Gordon. H. S., The Economic Theory of a Common Property Resource: The Fishery, Journal of Political Economy, 62 (1954):124-142.

Grafton, R. Q., R. Arnason, T. Bjorndal, D. Campbell, H. F. Campbell, C. W. Clark, R. Connor, D. P. Dupont, R. Hannesson, R. Hilborn, J. E. Kirkley, T. Kompas, D. E. Lane, G. R. Munro, S. Pascoe, D. Squires, S. I. Steinshamn, B. R. Turris, and Q. Weninger, 2006, Incentive-based approaches to sustainable fisheries, Canadian Journal of Fisheries and Aquatic Science, 63, 699-710.

Hahn, R. H., 1984, Market Power and Transferable Property Rights, Quarterly Journal of Economics, 99, 753-65.

Jovanovic, B., 1982, Selection and the Evolution of Industry, Econometrica, 50, 649-670.

Malueg, D. A., and A. J. Yates, 2009, Bilateral Oligopoly, Private Information and Pollution Permit Markets, Environmental and Resource Economics, 43, 553-572.

Montgomery, W. D., 1972, Markets in Licenses and Efficient Pollution Control Programs, Journal of Economic Theory, 5, 395-418.

Morris S. and H. S. Shin, 1998, Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks, American Economic Review, 88, 587-97.

Morris and Shin, 2002 Social Value of Public Information, American Economic Review, 92(5), 1521-1534

Morris and Shin, 2005; Amador and Weil, 2010; Vives, 1993, 1997, 2013; Angeletos and Pavan, 2007, 2009; Amador and Weil, 2010; Vives, 1993, 1997, 2013

Neary, J. P., and K. W. S. Roberts, 1980, The Theory of Household Behavior Under Rationing, European Economic Review, 13, 43-80.

Smith, V. L., Economics of Production from Natural Resources, American Economic Review, 58 (1968):409-431.

Stavins, R. N., Transactions Costs and Tradeable Permits, Journal of Environmental Economics and Management, 29, 133-148.

Vives, 1993, How fast do rational agents learn?, Review of Economic Studies, 60 (2), 329-347

Vives, 1997, Learning from others: A welfare analysis, Games and Economic Behavior, 20(2), 177-200

Vives, X., 2013, Endogenous Public Information and Welfare in Market Games, IESE Business School, working paper.

9 Appendix I

9.1 Equilibrium under complete information: Proof of Proposition 1

Under complete information about θ, the profit function for a firm with cost parameter θ_i assumes two different forms:

$$\pi(\theta_i, \theta) = \begin{cases} \frac{1}{2\lambda}(p - r(\theta, \theta^*) - \theta_i)^2, & \text{if for some } \theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon], r(\theta, \theta^*) > 0 \\ \frac{1}{2\lambda}(p - \theta_i)^2, & \text{if for all } \theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon], r(\theta, \theta^*) = 0 \end{cases}$$

As the equilibrium threshold θ^* is given by the solutions of $\pi(\theta^*, \theta) = \delta$, there are two different types of solutions to consider: (1) the solution of $\frac{1}{2\lambda}(p - \theta^*)^2 = \delta$ which applies when $r(\theta, \theta^*) = 0$ for all $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ and (2) the solution of $\frac{1}{2\lambda}(p - r(\theta, \theta^*) - \theta^*)^2 = \delta$ which applies when $r(\theta, \theta^*) > 0$ for some $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$.

9.1.1 $\delta < \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$

$\frac{1}{2\lambda}(p - \theta^*)^2 = \delta$ yields $\theta^* = p - \sqrt{2\lambda\delta}$ as the unique solution, if $\delta > 0$. For such a θ^* to be an equilibrium, it must be the case that $r(\theta, \theta^*) = 0$ for $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$. This implies that $r(\theta, \theta^*)$ attains a value of 0, at a value of $\theta < \theta^* - \varepsilon$, that is at a value of θ at which all firms are active and $A = S$. Given the form of the permit price function that applies when $A = S$, the value of θ at which this price falls to 0, is given by, $\hat{\theta} = p - \lambda Q$.

Hence $\theta^* = p - \sqrt{2\lambda\delta}$ is an equilibrium if the following conditions hold.

1. $\frac{1}{2\lambda}(\theta + \lambda Q - \theta_i)^2 > \delta$ for $\theta < p - \lambda Q$ and $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$.
2. $\frac{1}{2\lambda}(p - \theta_i)^2 > \delta$ for $\theta \geq p - \lambda Q$, $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$ and $\theta_i \leq p - \sqrt{2\lambda\delta}$.
3. $\frac{1}{2\lambda}(p - \theta_i)^2 < \delta$ for $\theta \geq p - \lambda Q$, $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$ and $\theta_i > p - \sqrt{2\lambda\delta}$.

Condition (1) implies, $\frac{1}{2\lambda}(\theta + \lambda Q - \theta - \varepsilon)^2 = \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2 > \delta$ which is true under the given parametric configuration. Since $\frac{1}{2\lambda}(p - \theta_i)^2$ is monotone decreasing in θ_i and has a zero at $\theta_i = p - \sqrt{2\lambda\delta}$, (2) and (3) are true.

Furthermore as $p - \theta^* > 0$, for all active firms with $\theta_i \leq \theta^*$, $p - \theta_i > 0$, implying every active firm produces strictly positive quantity in equilibrium.

9.1.2 $\delta \geq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$

When, $r(\theta, \theta^*) > 0$ for some $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$, the threshold θ^* is determined by $\frac{1}{2\lambda}\left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta^* + \frac{2\varepsilon\lambda Q}{\theta^* - (\theta - \varepsilon)}\right)^2 = \delta$ which yields

$$\theta^*(\theta) = (\theta - \varepsilon) \pm \sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}$$

Note that the form of the profit function applies only if $\theta - \varepsilon \leq \theta^*(\theta) \leq \theta + \varepsilon$. The first inequality implies that the relevant solution for θ^* is

$$\theta^*(\theta) = (\theta - \varepsilon) + \sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}$$

The second inequality implies $(\theta-\varepsilon)+\sqrt{2\lambda\delta+4\varepsilon\lambda Q}-\sqrt{2\lambda\delta}\leq\theta+\varepsilon$, which in turn implies $\delta\geq\frac{1}{2\lambda}(\lambda Q-\varepsilon)^2$, upon simplification and is satisfied by the parametric configuration assumed.

For the form of $\theta^*(\theta)$ under consideration, the admissible form for $r(\theta,\theta^*)$ is,

$$r(\theta,\theta^*)=\begin{cases} p+\varepsilon-\frac{\sqrt{2\lambda\delta+4\varepsilon\lambda Q}-\sqrt{2\lambda\delta}}{2}-\frac{2\varepsilon\lambda Q}{\sqrt{2\lambda\delta+4\varepsilon\lambda Q}-\sqrt{2\lambda\delta}}-\theta & \text{for } \theta<\hat{\theta} \\ 0, & \text{for } \theta>\hat{\theta} \end{cases}$$

where,

$$\begin{aligned}\hat{\theta} &= p+\varepsilon-\frac{\sqrt{2\lambda\delta+4\varepsilon\lambda Q}-\sqrt{2\lambda\delta}}{2}-\frac{2\varepsilon\lambda Q}{\sqrt{2\lambda\delta+4\varepsilon\lambda Q}-\sqrt{2\lambda\delta}} \\ &= p+\varepsilon-\sqrt{2\lambda\delta+4\varepsilon\lambda Q}\end{aligned}$$

upon simplification.

Hence the admissible threshold function is

$$\theta^*(\theta)=\begin{cases} (\theta-\varepsilon)+\sqrt{2\delta\lambda+4\varepsilon\lambda Q}-\sqrt{2\delta\lambda} & \text{for } \theta<\hat{\theta} \\ p-\sqrt{2\lambda\delta} & \text{for } \theta\geq\hat{\theta} \end{cases}$$

Note that as the condition $\theta^*(\theta)<\theta+\varepsilon$ implies that it is never profitable for all firms to be active simultaneously, in an equilibrium, the following must hold,

1. For $\theta<\hat{\theta}$

$$\begin{aligned} &\frac{1}{2\lambda}\left(\frac{\theta^*(\theta)+\theta-\varepsilon}{2}+\frac{2\varepsilon\lambda Q}{\theta^*(\theta)-(\theta-\varepsilon)}-\theta_i\right)^2>\delta \quad \text{for} \quad \theta_i\leq\theta^*(\theta) \\ &\frac{1}{2\lambda}\left(\frac{\theta^*(\theta)+\theta-\varepsilon}{2}+\frac{2\varepsilon\lambda Q}{\theta^*(\theta)-(\theta-\varepsilon)}-\theta_i\right)^2<\delta \quad \text{for} \quad \theta_i>\theta^*(\theta)\end{aligned}$$

2. For $\theta>\hat{\theta}$

$$\begin{aligned} &\frac{1}{2\lambda}(p-\theta_i)^2>\delta \quad \text{for} \quad \theta_i\leq\theta^*(\theta) \\ &\frac{1}{2\lambda}(p-\theta_i)^2<\delta \quad \text{for} \quad \theta_i>\theta^*(\theta)\end{aligned}$$

These conditions are satisfied because the profit functions are monotone decreasing in θ_i and has zeros at $\theta^*(\theta)$ for the relevant regions. Finally as, since $p-r(\theta,\theta^*)-\theta^*>0$, all active firms with $\theta_i\leq\theta^*$ earn strictly positive variable profits, $p-r(\theta,\theta^*)-\theta_i$, implying that every active firm produces strictly positive quantity in equilibrium.

10 Appendix II

10.1 Equilibrium under incomplete information - condition for strictly positive production

First we want to ensure that $p - r(\theta, \theta^*) - \theta_i > 0$ for the case when all firms are active - that is for all $\theta < \theta^* - \varepsilon$ and $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$. Under this situation,

$$p - r(\theta, \theta^*) - \theta_i = \theta + \lambda Q - \theta_i$$

It suffices to ensure that the required condition holds for the highest cost firm $\theta + \varepsilon$. Thus, we want $\theta + \lambda Q - \theta - \varepsilon > 0$ which yields $\lambda Q > \varepsilon$, the first part of Assumption 1.

We next want to ensure that $p - r(\theta, \theta^*) - \theta_i > 0$ for $\theta^* - \varepsilon \leq \theta \leq \theta^* + \varepsilon$ and $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$. Thus we want

$$\begin{array}{ll} \frac{\theta^*+\theta-\varepsilon}{2} + \frac{2\varepsilon}{\theta^*-(\theta-\varepsilon)} - \theta^* > 0 & \text{for} \quad r > 0 \\ p - \theta^* > 0 & \text{for} \quad r = 0 \end{array}$$

The first inequality reduces to the requirement, $\theta^* + \varepsilon - \theta) < \pm 2\sqrt{\varepsilon \lambda Q}$ on simplification. As we are looking at the case when $\theta \leq \theta^* + \varepsilon$, the requirement reduces to $\theta > \theta^* + \varepsilon - 2\sqrt{\varepsilon \lambda Q}$.

Note that $\lambda Q > \varepsilon$ implies $\sqrt{\varepsilon \lambda Q} > \varepsilon$ which in turn implies that $\theta^* + \varepsilon - 2\sqrt{\varepsilon \lambda Q} < \theta^* + \varepsilon - 2\varepsilon = \theta^* - \varepsilon$.

Since $\theta \geq \theta^* - \varepsilon$, the first inequality is automatically satisfied. Since under $\lambda Q > \varepsilon$, $p - r(\theta, \theta^*) - \theta^* > 0$ for $r > 0$, it is automatically true that $p - \theta^* > 0$ when $r = 0$.

10.2 Proof of Proposition 2:

There are multiple steps through which this Proposition will be proved.

Step 1.: Note that

$$\begin{gathered} \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda} > \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2 = \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{2\lambda} - \varepsilon Q \\ \Leftrightarrow \lambda Q > \frac{\varepsilon}{3} \end{gathered}$$

which is satisfied by Assumption 1. Hence the interval $(\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2, \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda}]$ is non-empty.

Step II. We next show that there is a unique solution θ^* that satisfy (14) and (15).

Consider the function,

$$\pi(k)=\frac{1}{2\varepsilon}\int_{k-\varepsilon}^{\hat{\theta}(k)}\frac{1}{2\lambda}\left(\frac{k+\theta-\varepsilon}{2}+\frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)}-k\right)^2 d\theta+\frac{1}{2\varepsilon}\int_{\hat{\theta}(k)}^{k+\varepsilon}\frac{1}{2\lambda}(p-k)^2 d\theta$$

where $\hat{\theta}(k)=p+\varepsilon-\sqrt{(p-k)^2+4\varepsilon\lambda Q}$. On simplification,

$$\begin{aligned}\pi(k)=&\frac{1}{4\lambda\varepsilon}\int_{k-\varepsilon}^{\hat{\theta}(k)}\left(\frac{4\left(\varepsilon\lambda Q\right)^2}{\left(\theta-(k+\varepsilon)\right)^2}+\frac{\left(\theta-(k+\varepsilon)\right)^2}{4}-2\varepsilon\lambda Q\right)d\theta\\&+\frac{1}{4\lambda\varepsilon}\left(p-k\right)^2\left[(k-p)+\sqrt{\left(p-k\right)^2+4\varepsilon\lambda Q}\right]\end{aligned}$$

A change of variable $z=\theta-(k+\varepsilon)$ allows us to evaluate the first integral. With this change of variable, the upper and lower limits of the integration are, respectively, $\hat{\theta}\left(k\right)-k-\varepsilon=(p-k)-\sqrt{\left(p-k\right)^2+4\varepsilon\lambda Q}$ and -2ε. Evaluating the integral using the new variable and then substituting the new variable back and simplifying, we have,

$$\pi\left(k\right)=\frac{1}{4\lambda\varepsilon}\left[\begin{array}{c}-\frac{4(\varepsilon\lambda Q)^2}{(p-k)-\sqrt{(p-k)^2+4\varepsilon\lambda Q}}-\frac{4(\varepsilon\lambda Q)^2}{2\varepsilon}+\frac{\left((p-k)-\sqrt{(p-k)^2+4\varepsilon\lambda Q}\right)^3}{12}\\+\frac{8\varepsilon^3}{12}-2\varepsilon\lambda Q\left[(p-k)-\sqrt{\left(p-k\right)^2+4\varepsilon\lambda Q}+2\varepsilon\right]\\-\left[(p-k)-\sqrt{\left(p-k\right)^2+4\varepsilon\lambda Q}\right]\left(p-k\right)^2\end{array}\right]\tag{24}$$

We try to show next that $\frac{d\pi(k)}{dk}<0$. A second change of variable helps us to do that. Define

$$x\equiv\sqrt{\left(p-k\right)^2+4\varepsilon\lambda Q}-(p-k)>0$$

and note that

$$\frac{dx}{dk}=1-\frac{p-k}{\sqrt{\left(p-k\right)^2+4\varepsilon\lambda Q}}>0$$

Further note that $(p-k)^2=\frac{x^2}{4}+\frac{(2\varepsilon\lambda Q)^2}{x^2}-2\varepsilon\lambda Q$.

With the second change of variable, $\pi(k)$ can be rewritten as

$$\pi\left(k,I_k\right)=\frac{1}{4\lambda\varepsilon}\left[\begin{array}{c}-\frac{4(\varepsilon\lambda Q)^2}{2\varepsilon}+\frac{8\varepsilon^3}{12}-2\varepsilon\lambda Q\left(2\varepsilon\right)\\+\frac{x^3}{6}+\frac{8(\varepsilon\lambda Q)^2}{x}\end{array}\right]$$

Thus, whether $\pi(k)$ is increasing or decreasing in k depends on whether it decreases or increases in x.

$$\frac{d\pi}{dx} = \frac{1}{4\lambda\varepsilon}\left[\frac{x^2}{2} - \frac{8(\varepsilon\lambda Q)^2}{x^2}\right]$$
$$= \frac{1}{4\lambda\varepsilon}\left(\frac{x}{\sqrt{2}} + \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right)\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right)$$

Thus the sign of $\frac{d\pi(k)}{dk}$ depends on the sign of $\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right)$. Substituting the expression for x back and simplifying, it is straightforward to check that so long as $(p-k) > 0$ (true for values of k we are interested in), $\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right) < 0$.

Thus $\frac{d\pi(k)}{dk} < 0$.

Hence, if the function $\pi(k)$ can be shown to be greater than δ for some k and less than δ for some k, it has a unique intersection with δ. We proceed to show it as follows.

Note that, for any given k, the following inequality is true.

$$\pi(k) \leq \frac{1}{2\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon} \frac{1}{2\lambda}(p-k)^2 \, d\theta = \frac{1}{2\lambda}(p-k)^2 \tag{25}$$

It is straightforward to verify that for $k = p - (\lambda Q - \varepsilon)$, $\frac{1}{2\lambda}(p-k)^2 = \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$. Hence, $\pi(k) \leq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2 < \delta$ for some value of k.

Next, note that for any given k, the following inequality is true, so long as $\theta + \lambda Q \leq p$.

$$\frac{1}{2\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon} \frac{1}{2\lambda}(\theta + \lambda Q - k)^2 \, d\theta \leq \pi(k) \tag{26}$$

The inequality is true by the following arguments. At $\theta = k - \varepsilon$, the expression $\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)}\right) = \theta + \lambda Q$. Both functions are increasing in θ, but the slope of $\theta + \lambda Q$ is 1. The slope of $\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)}\right)$ is given by $\left(\frac{1}{2} + \frac{2\varepsilon\lambda Q}{(k-(\theta-\varepsilon))^2}\right)$. As $\frac{2\varepsilon\lambda Q}{(k-(\theta-\varepsilon))^2} > \frac{4\varepsilon^2}{(k-(\theta-\varepsilon))^2}$ by Assumption 1 and $k - (\theta - \varepsilon) \leq 2\varepsilon$, the ratio $\frac{2\varepsilon\lambda Q}{(k-(\theta-\varepsilon))^2} > 1$ and hence $(\theta + \lambda Q - k) \leq \left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right)$ for $\theta \in [k-\varepsilon, k+\varepsilon]$, for any given k. Finally note that for any given k, the following is true, as long as $\theta + \lambda Q \leq p$.

$$(\theta + \lambda Q - k) \leq \left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right) \leq (p-k)$$

Hence, inequality (26) is true for $\theta + \lambda Q \leq p$.

Therefore, choose $k = p - \lambda Q - \varepsilon$. At $k = p - \lambda Q - \varepsilon$,

$$\pi(k) \geq \frac{1}{2\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon} \frac{1}{2\lambda}(\theta + \lambda Q - k)^2 \, d\theta = \frac{1}{12\lambda\varepsilon}\left[(\lambda Q + \varepsilon)^3 - \lambda Q - \varepsilon)^3\right] = \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda} \geq \delta$$

Thus the function $\pi(k)$ is greater than δ for some k and less than δ for some k. It therefore has a unique intersection $k = \theta^*$ with δ.

Although θ^* does not have a closed form, equations (25) and (26) are useful because they provide upper and lower bounds within which θ^* lie. Specifically, the above steps show us that, $p - \lambda Q - \varepsilon \leq \theta^*(\delta) \leq p - \lambda Q + \varepsilon$.

Step III. We next show that the switching strategy with the threshold θ^* is an equilibrium. The proof for this part involves showing that for any firm of type θ_i, $\pi(\theta_i, \theta^*) > \delta$ for $\theta_i < \theta^*$ and $\pi(\theta_i, \theta^*) < \delta$ for $\theta_i > \theta^*$. As a first step, we need to characterize the function $\pi(\theta_i, \theta^*)$ for different zones in which θ_i may lie, given the solution θ^*.

The following list characterizes and explains the individual profit function $\pi(\theta_i, \theta^*)$, for each zone in which θ_i may lie. In each case the condition that the profit function must satisfy for θ^* to be an equilibrium, is also provided.

1. $\theta_i < \theta^* - 2\varepsilon$

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta > \delta$$

   $\theta_i < \theta^* - 2\varepsilon \implies \theta_i + 2\varepsilon < \theta^*$. Hence, from the perspective of the θ_i-type, all possible types for any $\theta \in [\theta_i - \varepsilon, \theta_i - \varepsilon]$ are below the threshold θ^*. Thus, all firms are active for any $\theta \in [\theta_i - \varepsilon, \theta_i - \varepsilon]$, resulting in the above profit function. The expected profit from harvest for the θ_i type must be clearly higher than δ and the type will enter.

2. $\theta^* - 2\varepsilon < \theta_i < \hat{\theta}(\theta^*) - \varepsilon < \theta^*$.

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta^* - \varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\varepsilon} \int_{\theta^* - \varepsilon}^{\theta_i + \varepsilon} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* - (\theta - \varepsilon)} \right)^2 d\theta > \delta$$

   Under this case, not all firms are active for any $\theta \in [\theta_i - \varepsilon, \theta_i - \varepsilon]$. $\theta^* - 2\varepsilon < \theta_i \implies \theta^* < \theta_i + 2\varepsilon$. Hence if the actual $\theta = \theta_i + \varepsilon$, clearly some of the highest cost types will not enter. The expected profit function therefore has two parts depending on the values that θ can assume. Once again as $\theta_i < theta^*$, the expected profit must be greater than δ.

3. $\hat{\theta}(\theta^*) - \varepsilon < \theta_i < \theta^*$

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta^* - \varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\varepsilon} \int_{\theta^* - \varepsilon}^{\hat{\theta}(\theta^*)} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* - (\theta - \varepsilon)} \right)^2 d\theta$$
$$+ \frac{1}{4\lambda\varepsilon} \int_{\hat{\theta}(\theta^*)}^{\theta_i + \varepsilon} (p - \theta_i)^2 \, d\theta > \delta$$

   Similar arguments as above explain the first two components of the function. The third component is explained by the fact that if the actual $\theta > \hat{\theta}(\theta^*)$, the permit price falls to zero, assuming all other firms adopt the threshold of θ^*.

4. $\theta^* < \theta_i < \hat{\theta}(\theta^*) + \varepsilon < \theta^* + 2\varepsilon$

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\hat{\theta}(\theta^*)} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* - (\theta - \varepsilon)} \right)^2 d\theta + \frac{1}{4\lambda\varepsilon} \int_{\hat{\theta}(\theta^*)}^{\theta_i + \varepsilon} (p - \theta_i)^2 \, d\theta < \delta$$

The quota price is positive for some values of θ and zero for others, accounting for the two components. As $\theta_i > \theta^*$, production must be less profitable for the θ_i-type, compared to the outside option.

5. $\hat{\theta}(\theta^*) + \varepsilon < \theta_i < \theta^* + 2\varepsilon$

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (p - \theta_i)^2 \, d\theta < \delta$$

Finally, it can be checked that $\pi(\theta_i, \theta^*)$ is continuous in θ_i.

To prove the rest of the proposition, we need to show that the required inequality involving $\pi(\theta_i, \theta^*)$ and δ for each zone is satisfied.

1. When $\theta_i < \theta^* - 2\varepsilon$,

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta = \frac{1}{12\lambda\varepsilon} \left[(\lambda Q + \varepsilon)^3 - \lambda Q - \varepsilon)^3 \right] = \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda} > \delta$$

and the first inequality is satisfied.

2. When $\theta^* - 2\varepsilon < \theta_i < \hat{\theta}(\theta^*) - \varepsilon < \theta^*$, the argument immediately following equation (**??**) shows that for any θ_i and given θ^*,

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta^* - \varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\varepsilon} \int_{\theta^* - \varepsilon}^{\theta_i + \varepsilon} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* - (\theta - \varepsilon)} \right)^2 d\theta$$

$$\geq \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta > \delta$$

Thus the second inequality is also true.

3. Note that $\pi(\theta_i, \theta^*)$ is not only continuous but differentiable (in fact, twice differentiable) over the interval $\hat{\theta}(\theta^*) - \varepsilon < \theta_i < \theta^*$ as well. Because of continuity and our arguments in the previous case, at $\theta_i = \hat{\theta}(\theta^*) - \varepsilon$, $\pi(\theta_i, \theta^*) > \delta$. Moreover, as $\theta_i \longrightarrow \theta^*$, $\pi(\theta_i, \theta^*) \longrightarrow \pi(\theta^*) = \delta$. **Step II** of this proof shows that $\pi(\theta^*)$ is declining at $\theta_i = \theta^*$. Hence the slopes of the two functions must also converge as $\theta_i \longrightarrow \theta^*$ and in particular $\pi(\theta_i, \theta^*)$ must be declining at $\theta_i = \theta^*$. These statements taken together imply that $\pi(\theta_i, \theta^*)$ must have at least one stationary point that is a maximum in this interval.

We therefore check the roots of the derivative of $\pi(\theta_i, \theta^*)$ with respect to θ_i.

The derivative of the first term of $\pi(\theta_i, \theta^*)$ with respect to θ_i is given by (using Leibnitz's rule),

$$\begin{aligned}
&\frac{-1}{2\lambda\varepsilon}\int_{\theta_i-\varepsilon}^{\theta^*-\varepsilon}(\theta-\theta_i+\lambda Q)\,d\theta-(\lambda Q-\varepsilon)^2\\
&=-(\theta^*-\theta_i+\lambda Q-\varepsilon)^2+(\lambda Q-\varepsilon)^2-(\lambda Q-\varepsilon)^2\\
&=-(\theta^*-\theta_i+\lambda Q-\varepsilon)^2
\end{aligned}$$

The derivative of the second term is given by,

$$\begin{aligned}
&\frac{-1}{2\lambda\varepsilon}\int_{\theta^*-\varepsilon}^{\hat{\theta}(\theta^*)}\left(\frac{\theta^*+\theta-\varepsilon}{2}-\theta_i+\lambda\frac{2\varepsilon Q}{\theta^*-(\theta-\varepsilon)}\right)d\theta\\
&=2(\theta^*-\varepsilon)^2-\frac{\left(\theta^*+\hat{\theta}-\varepsilon\right)^2}{2}+4\varepsilon\lambda Q\left(\ln\left[\theta^*+\varepsilon-\hat{\theta}\right]-\ln[2\varepsilon]\right)+2\theta_i\left(\hat{\theta}(\theta^*)-(\theta^*-\varepsilon)\right)
\end{aligned}$$

The derivative of the third term is given by

$$\frac{1}{4\lambda\varepsilon}\left[3(p-\theta_i)^2-2(p-\theta_i)\sqrt{(p-\theta^*)^2+4\varepsilon\lambda Q}\right]$$

These terms of the derivatives can be combined to get

$$\frac{1}{2\lambda\varepsilon}\left[\theta_i^2-2\left(p-\frac{\lambda Q+\varepsilon}{2}\right)\theta_i+\Omega[\theta^*]\right] \tag{27}$$

where

$$\begin{aligned}
\Omega[\theta^*]\equiv{}&(\theta^*-\varepsilon)^2-\frac{\left(\hat{\theta}(\theta^*)+\theta^*-\varepsilon\right)^2}{4}+2\varepsilon\lambda Q\log\left[1-\frac{\hat{\theta}(\theta^*)-(\theta^*-\varepsilon)}{2\varepsilon}\right]\\
&+\frac{p^2-(\lambda Q+\theta^*-\varepsilon)^2}{2}+p\left(\hat{\theta}(\theta^*)-\varepsilon\right)
\end{aligned}$$

Derivative (27) has two roots given by

$$\theta_{1,2}^R(\theta^*)\equiv p-\frac{\lambda Q+\varepsilon}{2}\pm\sqrt{\left(p-\frac{\lambda Q+\varepsilon}{2}\right)^2-\Omega[\theta^*]},$$

Note that both roots cannot be less than θ^* because of the following contradiction.

$$\theta_1^R=\theta_{1,2}^R(\theta^*)\equiv p-\frac{\lambda Q+\varepsilon}{2}-\sqrt{\left(p-\frac{\lambda Q+\varepsilon}{2}\right)^2-\Omega[\theta^*]}<\theta^*\Longrightarrow 2\left(p-\frac{\lambda Q+\varepsilon}{2}\right)\theta^*-\theta^{*2}>\Omega[\theta^*]$$

$$\theta_2^R = \theta_{1,2}^R(\theta^*) \equiv p - \frac{\lambda Q + \varepsilon}{2} + \sqrt{\left(p - \frac{\lambda Q + \varepsilon}{2}\right)^2 - \Omega[\theta^*]} < \theta^* \Longrightarrow 2\left(p - \frac{\lambda Q + \varepsilon}{2}\right)\theta^* - \theta^{*2} < \Omega[\theta^*]$$

but $\theta_2^R > \theta_1^R$.

As one of the roots must be less than θ^* for $\pi(\theta_i, \theta^*)$ to be declining at $\theta_i = \theta^*$, $\theta_1^R < \theta^*$. Thus $\pi(\theta_i, \theta^*)$ has only one stationary point in the interval which is a maximum and assumes a value of δ at $\theta_i = \theta^*$. Hence $\pi(\theta_i, \theta^*) > \delta$ over the interval.

4. For $\theta^* < \theta_i < \hat{\theta}(\theta^*) + \varepsilon < \theta^* + 2\varepsilon$, as in the previous case, the function $\pi(\theta_i, \theta^*)$ is twice differentiable. The derivative of the profit function is $\frac{1}{4\lambda\varepsilon}$ times the expression,

$$(p - \theta_i)^2 - \left[\begin{array}{c} \left(\frac{\theta^* - \varepsilon + \theta_i - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* + \varepsilon - (\theta_i - \varepsilon)}\right)^2 \\ +2\left(\int_{\theta_i - \varepsilon}^{\hat{\theta}(\theta^*)} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \frac{2\lambda\varepsilon Q}{\theta^* - (\theta - \varepsilon)}\right) d\theta + 2\int_{\hat{\theta}(\theta^*)}^{\theta_i + \varepsilon} (p - \theta_i)\, d\theta\right) \end{array}\right] \tag{28}$$

where the term within square brackets is positive. In particular, note that

$$\left(\frac{\theta^* - \varepsilon + \theta_i - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* + \varepsilon - (\theta_i - \varepsilon)} > 0\right) \quad \text{for} \quad \theta \in [\theta_i - \varepsilon, \hat{\theta}(\theta^*)]$$

a property that we use in the very next step.

The second order cross partial derivative, $\frac{\partial^2 \pi(\theta_i, \theta^*)}{\partial \theta^* \partial \theta_i}$, is given by $\frac{1}{4\lambda\varepsilon}$ times the expression,

$$-\left[\begin{array}{c} \left(\frac{\theta^* - \varepsilon + \theta_i - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* + \varepsilon - (\theta_i - \varepsilon)}\right)\left(1 - \frac{4\varepsilon\lambda Q}{(\theta^* + 2\varepsilon - \theta_i)^2}\right) \\ \int_{\theta_i - \varepsilon}^{\hat{\theta}(\theta^*)} \left(1 - \frac{4\varepsilon\lambda Q}{(\theta^* + \varepsilon - \theta)^2}\right) d\theta \end{array}\right]$$

As $\lambda Q > \varepsilon$, $4\varepsilon\lambda Q > 4\varepsilon^2 \geq (\theta^* + 2\varepsilon - \theta_i)^2$, the first term within square brackets is negative. Similarly, as $\lambda Q > \varepsilon$, $4\varepsilon\lambda Q > 4\varepsilon^2 \geq (\theta^* + \varepsilon - \theta)^2$, the second term within square brackets is also negative. Hence $\frac{\partial^2 \pi(\theta_i, \theta^*)}{\partial \theta^* \partial \theta_i} \geq 0$ for this region, and the slope of $\pi(\theta_i, \theta^*)$ has the single crossing property. In particular this implies that if for a given θ^* and some θ_i such that $\theta^* = \theta_i$, the slope of $\pi(\theta_i, \theta^*)$ is negative, then the slope cannot be positive for higher values of θ_i for which $\theta^* < \theta_i$.

Finally, it can be checked that for the (Step II) solution θ^*, $\theta_i = \theta^*$, the slope of the region 4 and the region 3 profit functions are identical and negative. Hence, the required inequality is satisfied.

5. For, $\hat{\theta}(\theta^*) + \varepsilon < \theta_i < \theta^* + 2\varepsilon$, as the profit function equals

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (p - \theta_i)^2\, d\theta = \frac{1}{4\lambda\varepsilon} (p - \theta_i)^2$$

the derivative is given by

$$-\frac{1}{2\lambda} (p - \theta_i)$$

which is negative for admissible values of p. This proves the proposition.

10.3 Proof of Proposition 3

As under the previous scenario, the Proposition will be proved through multiple steps.

Step I. We first show that there is a unique solution θ^* that satisfy (16) and (17).

This is straightforward, as the unique solution of

$$\frac{1}{2\lambda}(p-\theta^*)^2=\delta$$

that satisfies the condition $\delta \leq \frac{1}{2\lambda}(\lambda Q-\varepsilon)^2$ and hence condition (17), is $\theta^*=p-\sqrt{2\lambda\delta}$.

Step II We show that the solution $\theta^*=p-\sqrt{2\lambda\delta}$ is an equilibrium. As we are looking for an equilibrium under which $r(\theta,\theta^*)=0$ for all $\theta\in[\theta^*-\varepsilon,\theta^*+\varepsilon]$, at the value of θ at which the equilibrium permit price falls to zero, all firms are active. Hence the value of θ at which the equilibrium permit price equals zero is given by $p-\theta-\lambda Q=0$. Denote by $\hat{\theta}=p-\lambda Q$, this value and note that it is independent of θ^*.

The individual profit function takes on three different forms depending on three zones in which θ_i may lie. We discuss these forms and the condition that each form must satisfy for θ^* to be equilibrium below. We also simultaneously show that these conditions are met.

1. For $\theta_i<p-\sqrt{2\lambda\delta}-2\varepsilon$, all firms participate for all possible values of θ. Hence individual profit function must satisfy the condition,

$$\pi(\theta_i,\theta^*)=\frac{1}{4\lambda\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}(\theta-\theta_i+\lambda Q)^2\,d\theta>\delta$$

   Note that

$$\begin{aligned}\pi(\theta_i,\theta^*)&=\frac{1}{4\lambda\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}(\theta-\theta_i+\lambda Q)^2\,d\theta\\&=\frac{1}{12\lambda\varepsilon}\left[(\lambda Q+\varepsilon)^3-(\lambda Q-\varepsilon)^3\right]\\&=\frac{1}{12\lambda\varepsilon}(2\varepsilon)\left((\lambda Q)^2-\varepsilon^2+2(\lambda Q)^2+2\varepsilon^2\right)\\&=\frac{\lambda Q^2}{2}+\frac{\varepsilon^2}{6\lambda}>\frac{1}{2\lambda}(\lambda Q-\varepsilon)^2\geq\delta\end{aligned}$$

   Hence the condition is satisfied.

2. For $p-\sqrt{2\lambda\delta}-2\varepsilon\leq\theta_i<p-\lambda Q+\varepsilon\leq\theta^*$, depending upon the value of θ, we may either have all firms active and the permit price is strictly positive, or else permit price is zero. Hence the profit function and the required condition are

$$\pi(\theta_i,\theta^*) = \frac{1}{4\lambda\varepsilon}\int_{\theta_i-\varepsilon}^{p-\lambda Q}(\theta-\theta_i+\lambda Q)^2\,d\theta + \frac{1}{4\lambda\varepsilon}\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)^2\,d\theta > \delta$$

Note that,

$$\begin{aligned}
\pi(\theta_i,\theta^*) &= \frac{1}{4\lambda\varepsilon}\int_{\theta_i-\varepsilon}^{p-\lambda Q}(\theta-\theta_i+\lambda Q)^2\,d\theta + \frac{1}{4\lambda\varepsilon}\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)^2\,d\theta \\
&= \frac{1}{4\lambda\varepsilon}\left[\frac{(p-\theta_i)^3}{3} - \frac{(\lambda Q-\varepsilon)^3}{3} + (p-\theta_i)^2(\theta_i+\varepsilon-p+\lambda Q)\right]
\end{aligned}$$

The above equals $\frac{\lambda Q^2}{2}+\frac{\varepsilon^2}{6\lambda}$ when $\theta_i = p-\lambda Q-\varepsilon$, and $\frac{1}{2\lambda}(\lambda Q-\varepsilon)^2 \geq \delta$ when $\theta_i = p-\lambda Q+\varepsilon$, The derivative of $\pi(\theta_i, I_{\theta^*})$ with respect to θ_i is $\frac{1}{4\lambda\varepsilon}$ times

$$\begin{aligned}
&-(\lambda Q-\varepsilon)^2 + (p-\theta_i)^2 - 2\int_{\theta_i-\varepsilon}^{p-\lambda Q}(\theta-\theta_i+\lambda Q)\,d\theta - 2\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)\,d\theta \\
&= -(\lambda Q-\varepsilon)^2 + (p-\theta_i)^2 - (p-\theta_i)^2 + (\lambda Q-\varepsilon)^2 - 2\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)\,d\theta \\
&= -2\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)\,d\theta < 0
\end{aligned}$$

Hence, it is strictly declining in θ_i through the range under consideration and therefore the condition is satisfied.

3. When $\theta_i > p-\lambda Q+\varepsilon \geq \theta^*$, we must have

$$\pi(\theta_i,\theta^*) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}\frac{1}{2\lambda}(p-\theta_i)^2 < \delta$$

Since $\frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}\frac{1}{2\lambda}(p-\theta_i)^2 = \frac{1}{2\lambda}(p-\theta_i)^2$, and the latter function is strictly declining in θ_i, the condition is satisfied, because at $\theta_i=\theta^*$, $\frac{1}{2\lambda}(p-\theta_i)^2=\delta$.

Thus the Proposition is proved.

11 Appendix III

The function is characterized below for two different cases - (1) $\hat{\theta}(\delta) \leq \theta^{*i}(\delta)+\epsilon$ which holds when $2\lambda\delta < (p-\theta^{*i}(\delta))^2 \leq 2\lambda\delta + 4\varepsilon\lambda Q$ and (2) $\theta^{*i}(\delta)+\epsilon \leq \hat{\theta}(\delta)$ which holds when $2\lambda\delta < 2\lambda\delta + 4\varepsilon\lambda Q < (p-\theta^{*i}(\delta))^2$.

11.1 Case 1: $\hat{\theta}(\delta) \leq \theta^{*i}(\delta) + \varepsilon$

The function, $(\alpha^i(\delta) - \alpha^f(\delta))$, has the following form,

A. For $\theta^l \leq \theta \leq \theta^{*i} - \varepsilon$, as $\alpha^i(\delta) = 1$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = 1 - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon} \tag{29}$$

which is constant in θ.

B. For $\theta^{*i} - \varepsilon \leq \theta \leq \hat{\theta}(\delta) \leq \theta^{*i} + \varepsilon$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{\theta^{*i} - (\theta - \varepsilon)}{2\varepsilon} - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon} \tag{30}$$

which is linear and decreasing in θ.

C. For $\hat{\theta}(\delta) \leq \theta \leq \theta^{*i} + \varepsilon$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{\theta^{*i} - (p - \sqrt{2\lambda\delta})}{2\varepsilon} \tag{31}$$

after due simplification and is a negative constant in θ.

D. For $\theta \leq \theta^{*i} + \varepsilon \leq \theta \leq \theta^h$, as $\alpha^i(\delta) = 0$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{(\theta - \varepsilon) - (p - \sqrt{2\lambda\delta})}{2\varepsilon} \tag{32}$$

after due simplification and is linear and increasing in θ.

E. For $\theta > \theta^h$, the active mass is zero under both incomplete and full information. Hence, $(\alpha^i(\delta) - \alpha^f(\delta)) = 0$

Figure 1 plots the difference, $\alpha^i - \alpha^f$ for Case 1, for a given value of δ and θ^l.

11.2 Case 2: $\theta^{*i}(\delta) + \varepsilon \leq \hat{\theta}(\delta)$

The function, $(\alpha^i(\delta) - \alpha^f(\delta))$, has the following form,

A. For $\theta^l \leq \theta \leq \theta^{*i} - \varepsilon$, as $\alpha^i(\delta) = 1$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = 1 - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon} \tag{33}$$

which is constant in θ.

B. For $\theta^{*i} - \varepsilon \le \theta \le \theta^{*i} + \varepsilon \le \hat{\theta}(\delta)$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{\theta^{*i} - (\theta - \varepsilon)}{2\varepsilon} - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon} \tag{34}$$

which is linear and decreasing in θ.

C. For $\theta^{*i} + \varepsilon \le \theta \le \hat{\theta}(\delta)$, as $\alpha^i(\delta) = 0$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = -\left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right) \tag{35}$$

which is negative and constant in θ.

D. For $\hat{\theta}(\delta) \le \theta \le \theta^h$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{(\theta - \varepsilon) - (p - \sqrt{2\lambda\delta})}{2\varepsilon} \tag{36}$$

which is linear and increasing in θ.

E. For $\theta > \theta^h$, as before, $(\alpha^i(\delta) - \alpha^f(\delta)) = 0$

Figure 2 plots the difference, $\alpha^i - \alpha^f$ for Case 1, for a given value of δ and θ^l.

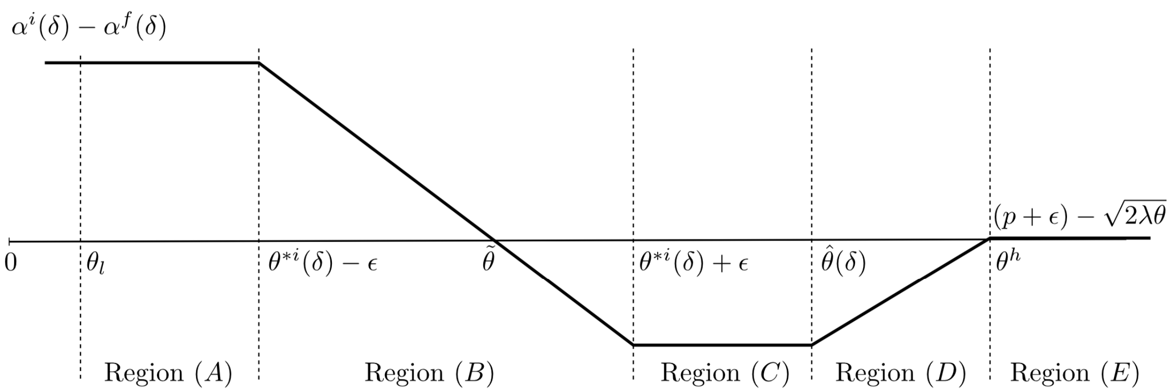

Figure 2: $(\alpha^i(\delta) - \alpha^f(\delta))$: Case 2

Thus under both cases, the function has the same form over the first two and the last zones.

We first derive the expression for the expected difference in active mass, $E(\alpha^i - \alpha^f)$, for the Case 5.1. Note that this expected difference is the definite integral of the function, $(\alpha^i - \alpha^f)$, over the domain $[\theta^l, \theta^h]$. As depicted in the figures, the function attains zero at some value of $\theta = \tilde{\theta} \in [\theta^{*i}(\delta) - \varepsilon, \hat{\theta}(\delta)]$. Using the functional form for this domain, we obtain,

$$\tilde{\theta} = \hat{\theta} - (p - \sqrt{2\lambda\delta} - \theta^{*i}(\delta))$$

The height of the function at $\theta = \theta^{*i}(\delta) - \varepsilon$ is $1 - \left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right)$ and the height at $\theta = \hat{\theta}$ is $\frac{(p - \sqrt{2\lambda\delta} - \theta^{*i}(\delta))}{2\varepsilon}$.

It is straightforward to show that the negative area enclosed by the function is given by,

$$\left(p - \sqrt{2\lambda\delta} - \theta^{*i}(\delta)\right)\left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right)$$

Similarly, the positive area supported by $\tilde{\theta} - (\theta^{*i}(\delta) - \varepsilon)$ is,

$$\left(\frac{(2\varepsilon - (\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}))^2}{4\varepsilon}\right)$$

And the positive area supported by $((\theta^{*i}(\delta) - \varepsilon) - \theta^l)$ is

$$\left((\theta^{*i}(\delta) - \varepsilon) - \theta^l\right)\left(1 - \left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right)\right)$$

After due simplification, the net area has the required expression presented by equation (20).

Following the same steps, it can be shown that the expression has the same form for Case 5.2. Since $\delta^l = \frac{(\lambda Q - \varepsilon)^2}{2\lambda}$, for $\delta = \delta^l$, $\theta^{*i}(\delta) = p - \sqrt{2\lambda\delta}$ and $\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda} = 2\varepsilon$. Upon substitution, the statements in Proposition 4 follows.

Note: Since the main result of this section deals with the relationship between $E(\alpha^i(\delta) - \alpha^f(\delta))$ and δ, we begin by discussing how the function $\alpha^i(\delta) - \alpha^f(\delta)$ is affected by changes in δ. The height of the function at the point $\theta^{*i}(\delta) - \varepsilon$ is given by $\left(1 - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right)$, a magnitude that increases with δ. The equilibrium values of $\theta^{*i}(\delta)$ and $\hat{\theta}(\delta)$ are both inversely related to δ, implying that all the cardinal points of the function, $\theta^{*i}(\delta) - \varepsilon$, $\theta^{*i}(\delta) + \varepsilon$, $\hat{\theta}(\delta)$, $(p + \varepsilon - \sqrt{2\delta\lambda})$ and $\tilde{\theta}$ shift left, closer towards the fixed θ^l. The height of the function at $\hat{\theta}(\delta)$ is given by $\frac{\theta^{*i}(\delta) + (\sqrt{2\lambda\delta} - p}{2\varepsilon}$ under Case 1. It is not clear how this height varies with δ as the first two terms in the numerator are affected in opposite ways. The same height under Case 2 has a different expression that is clearly increasing in δ. The function $\alpha^i(\delta) - \alpha^f(\delta)$ is thus affected in complex ways by changes in δ.

The expression (20) is the simplified form of the sum of the following two components,

$$\begin{aligned} E(\alpha^i - \alpha^f) &= (\theta^{*i}(\delta) - \varepsilon - \theta^l)\left(1 - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right) \\ &+ \left(\lambda Q + \varepsilon - (p - \theta^{*i}(\delta) + 2\varepsilon)\left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right)\right) \end{aligned}$$

The first component represents the positive area supported by $\theta \in [\theta^l, \theta^{*i}(\delta) - \varepsilon]$ and the second component represents the net positive area with support $\theta \in [\theta^{*i}(\delta) - \varepsilon, \theta^h]$. It is clear from the previous discussions that there is no obvious reason why the sum of the two components must be positive for any given value of δ and θ^l. We conjecture however that for sufficiently low values of δ and appropriate values of θ^l, the sum is positive - implying that for these values of δ and θ^l, the active mass under incomplete information is bigger than that under full information, in an expected sense. Proposition 4 shows that the conjecture is correct.

12 Appendix IV

12.1 Proof of Proposition 5:

As in the case of $\beta = 0$, the Proposition will be proved through multiple steps.

Step 1.: To show that the interval $(\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}]$ is non-empty under appropriate restrictions on the parameters, it suffices to show that the following inequality is true for appropriate restrictions on the parameters.

$$\left(\frac{2\varepsilon\lambda Q}{2\varepsilon - \beta} - \frac{2\varepsilon - \beta}{2}\right)^2 < (\lambda Q + \beta)^2$$

Above inequality implies

$$\left(\frac{2\varepsilon\lambda Q}{2\varepsilon - \beta} - \frac{2\varepsilon - \beta}{2}\right) < (\lambda Q + \beta)$$

which on simplification turns out to be

$$\lambda Q < \frac{(2\varepsilon - \beta)(2\varepsilon - \beta)}{2\beta}$$

Thus for any value of $\beta \in (0, 2\varepsilon]$, in particular for any value of $\beta \in (0, \varepsilon]$, there exists an upper bound on λQ, for which the above inequality is satisfied and the desired interval is non-empty. For example, when $\beta = \varepsilon$, the desired interval is non-empty if $\lambda Q < \frac{3}{2}\varepsilon$.

Step II. We next show that there is a unique solution μ^* that satisfy (22) and (23).

Consider the function,

$$\pi(k) = \frac{1}{2\varepsilon}\int_{k+\beta-\varepsilon}^{\hat{\theta}(k)} \frac{1}{2\lambda}\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right)^2 d\theta + \frac{1}{2\varepsilon}\int_{\hat{\theta}(k)}^{k+\beta+\varepsilon} \frac{1}{2\lambda}(p-k)^2 d\theta$$

where $\hat{\theta}(k) = p + \varepsilon - \sqrt{(p-k)^2 + 4\varepsilon\lambda Q}$. On simplification,

$$
\pi(k) = \frac{1}{4\lambda\varepsilon} \int_{k+\beta-\varepsilon}^{\hat{\theta}(k)} \left(\frac{4\left(\varepsilon\lambda Q\right)^2}{\left(\theta-(k+\varepsilon)\right)^2} + \frac{\left(\theta-(k+\varepsilon)\right)^2}{4} - 2\varepsilon\lambda Q \right) d\theta
$$
$$
+ \frac{1}{4\lambda\varepsilon}(p-k)^2 \left[(k+\beta-p) + \sqrt{(p-k)^2 + 4\varepsilon\lambda Q} \right]
$$

As before, a change of variable $z = \theta - (k+\varepsilon)$ allows us to evaluate the first integral. With this change of variable, the upper and lower limits of the integration are, respectively, $\hat{\theta}(k) - k - \varepsilon = (p-k) - \sqrt{(p-k)^2 + 4\varepsilon\lambda Q}$ and $\beta - 2\varepsilon$. Evaluating the integral using the new variable and then substituting the new variable back and simplifying, we have,

$$
\pi(k) = \frac{1}{4\lambda\varepsilon} \left[\begin{array}{c} -\frac{4(\varepsilon\lambda Q)^2}{(p-k)-\sqrt{(p-k)^2+4\varepsilon\lambda Q}} - \frac{4(\varepsilon\lambda Q)^2}{2\varepsilon} + \frac{\left((p-k)-\sqrt{(p-k)^2+4\varepsilon\lambda Q}\right)^3}{12} \\ +\frac{(2\varepsilon-\beta)^3}{12} - 2\varepsilon\lambda Q \left[(p-k) - \sqrt{(p-k)^2 + 4\varepsilon\lambda Q} + 2\varepsilon - \beta \right] \\ -\left[(p-k) - \beta - \sqrt{(p-k)^2 + 4\varepsilon\lambda Q} \right] (p-k)^2 \end{array} \right] \tag{37}
$$

We try to show next that $\frac{d\pi(k)}{dk} < 0$. A second change of variable helps us to do that. Define

$$
x \equiv \sqrt{(p-k)^2 + 4\varepsilon\lambda Q} - (p-k) > 0
$$

and note that

$$
\frac{dx}{dk} = 1 - \frac{p-k}{\sqrt{(p-k)^2 + 4\varepsilon\lambda Q}} > 0
$$

Further note that $(p-k)^2 = \frac{x^2}{4} + \frac{(2\varepsilon\lambda Q)^2}{x^2} - 2\varepsilon\lambda Q$.

With the second change of variable, $\pi(k)$ can be rewritten as

$$
\pi(k) = \frac{1}{4\lambda\varepsilon} \left[\begin{array}{c} -\frac{4(\varepsilon\lambda Q)^2}{2\varepsilon-\beta} + \frac{(2\varepsilon-\beta)^3}{12} - 2\varepsilon\lambda Q\,(2\varepsilon-\beta) \\ +\frac{x^3}{6} + \frac{8(\varepsilon\lambda Q)^2}{x} + \beta(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x})^2 \end{array} \right]
$$

Thus, whether $\pi(k)$ is increasing or decreasing in k depends on whether it decreases or increases in x.

$$
\frac{d\pi}{dx} = \frac{1}{4\lambda\varepsilon} \left[\frac{x^2}{2} - \frac{8(\varepsilon\lambda Q)^2}{x^2} \right] + \beta \left(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x} \right) \left(\frac{1}{2} + \frac{2\varepsilon\lambda Q}{x} \right)
$$
$$
= \frac{1}{4\lambda\varepsilon} \left(\frac{x}{\sqrt{2}} + \frac{2\sqrt{2}\varepsilon\lambda Q}{x} \right) \left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x} \right) + \beta \left(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x} \right) \left(\frac{1}{2} + \frac{2\varepsilon\lambda Q}{x} \right)
$$

Thus the sign of $\frac{d\pi(k)}{dk}$ depends on the signs of the terms, $\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right)$ and $\left(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x}\right)$.

Substituting the expression for x back and simplifying, it is straightforward to check that so long as $(p-k) > 0$ (true for values of k we are interested in),

$$\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right) = -\sqrt{2}(p-k) < 0$$

and

$$\left(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x}\right) = -(p-k) < 0$$

Thus $\frac{d\pi(k)}{dk} < 0$.

We next need to show that the function $\pi(k)$ is greater than δ for some k and less than δ for some k.

As before, for any given k, the following inequality is true.

$$\pi(k) \leq \frac{1}{2\varepsilon}\int_{k+\beta-\varepsilon}^{k+\beta+\varepsilon} \frac{1}{2\lambda}(p-k)^2\, d\theta = \frac{1}{2\lambda}(p-k)^2 \tag{38}$$

Moreover, for $k = p - (\lambda Q - \varepsilon)$, $\pi(k) \leq \frac{1}{2\lambda}(p-k)^2 = \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$. Since, $\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2 < (\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2 < \delta$ for $0 < \beta \leq 2\varepsilon$, $\pi(k) < \delta$ for some value of k.

Similarly, for any given k, the following inequality is true for $\theta \leq p - \lambda Q$.

$$\frac{1}{2\varepsilon}\int_{k+\beta-\varepsilon}^{k+\beta+\varepsilon} \frac{1}{2\lambda}(\theta + \lambda Q - k)^2\, d\theta \leq \pi(k) \tag{39}$$

The inequality is true by the following arguments. At $\theta = k + \beta - \varepsilon$, the expressions within integral signs have following values:

$$\theta + \lambda Q - k = \lambda Q + \beta - \varepsilon$$

$$\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right) = \left(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2}\right)$$

Note that,

$$\left(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2}\right) - (\lambda Q + \beta - \varepsilon) = \frac{\beta}{2}\left(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - 1\right) > 0,$$

since $\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} > 1$.

Hence, for $\theta = k + \beta - \varepsilon$,

$$\theta + \lambda Q - k < \left(\frac{k + \theta - \varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k - (\theta - \varepsilon)} \right)$$

Both functions are increasing in θ, but the slope of $\theta + \lambda Q$ is 1 and following the same steps as in Proposition 2, we can show that the slope of the RHS expression is greater than 1.

Hence, for any given k and $\theta \leq p - \lambda Q$, inequality (39) is true.

Thus, for $k = p - \lambda Q - \varepsilon$,

$$\pi(k) \geq \frac{1}{2\varepsilon} \int_{k-\varepsilon}^{k+\varepsilon} \frac{1}{2\lambda} (\theta + \lambda Q - k)^2 \, d\theta = \frac{1}{12\lambda\varepsilon} \left[(\lambda Q + \beta + \varepsilon)^3 - (\lambda Q + \beta - \varepsilon)^3 \right] = \frac{(\lambda Q + \beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda} \geq \delta$$

Hence $\pi(k)$ has a unique intersection $k = \mu^*$ with δ.

Step III. We next show that the switching strategy with the threshold μ^* is an equilibrium. As the proof is very similar to the proof in Proposition 2, we shorten or skip altogether the parts that are identical and focus only on those where there are differences. As before, we need to show that for any firm of type θ_i, $\pi(\theta_i, \mu^*) > \delta$ for $\theta_i < \mu^*$ and $\pi(\theta_i, \mu^*) < \delta$ for $\theta_i > \mu^*$.

The following list characterizes the individual profit function $\pi(\theta_i, \mu^*)$, for each zone in which θ_i may lie and provides the condition that the profit function must satisfy, for θ^* to be an equilibrium, in each of these zones. The rational for the form of the profit function for each zone is identical to that for the no-bias ($\beta = 0$) case and is therefore omitted. The only exception is zone 4 below which is new to $\beta > 0$.

1. $\theta_i < \mu^* - \beta - 2\varepsilon$

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta > \delta$$

2. $\mu^* - \beta - 2\varepsilon < \theta_i < \hat{\theta}(\mu^*) - \beta - \varepsilon < \mu^* - \beta$.

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \left[\int_{\theta_i+\beta-\varepsilon}^{\mu^*-\varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \int_{\mu^*-\varepsilon}^{\theta_i+\beta+\varepsilon} \left(\frac{\mu^* + \theta - \varepsilon}{2} - \theta_i + \frac{2\varepsilon\lambda Q}{\mu^* - (\theta - \varepsilon)} \right)^2 d\theta \right] > \delta$$

3. $\hat{\theta}(\mu^*) - \beta - \varepsilon < \theta_i < \mu^* - \beta$

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon}\left[\int_{\theta_i+\beta-\varepsilon}^{\mu^*-\varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \int_{\mu^*-\varepsilon}^{\hat{\theta}(\mu^*)} \left(\frac{\mu^* + \theta - \varepsilon}{2} - \theta_i + \frac{2\varepsilon\lambda Q}{\mu^* - (\theta - \varepsilon)}\right)^2 d\theta \right.$$
$$\left. + \int_{\hat{\theta}(\mu^*)}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2 \, d\theta \right] > \delta$$

4. $\mu^* - \beta < \theta_i < \mu^*$.

$$\pi(\theta_i, \mu^*) = \int_{\theta_i+\beta-\varepsilon}^{\hat{\theta}(\mu^*)} \left(\frac{\mu^* + \theta - \varepsilon}{2} - \theta_i + \frac{2\varepsilon\lambda Q}{\mu^* - (\theta - \varepsilon)}\right)^2 d\theta + \int_{\hat{\theta}(\mu^*)}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2 \, d\theta > \delta$$

   Since $\theta \in [\theta_i + \beta - \varepsilon, \theta_i + \beta + \varepsilon]$, $\mu^* - \beta < \theta_i \Longrightarrow \mu * - \varepsilon < \theta$ for all possible values of θ. Hence, it is never the case that all firms are active and this explains the form of the profit function.

5. $\mu^* < \theta_i < \hat{\theta}(\mu^*) - \beta + \varepsilon$

$$\pi(\theta_i, \mu^*) = \int_{\theta_i+\beta-\varepsilon}^{\hat{\theta}(\mu^*)} \left(\frac{\mu^* + \theta - \varepsilon}{2} - \theta_i + \frac{2\varepsilon\lambda Q}{\mu^* - (\theta - \varepsilon)}\right)^2 d\theta + \int_{\hat{\theta}(\mu^*)}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2 \, d\theta < \delta$$

6. $\hat{\theta}(\mu^*) - \beta + \varepsilon < \theta_i$

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2 \, d\theta < \delta$$

To prove the rest of the proposition, we need to show that the required inequality involving $\pi(\theta_i, \mu^*)$ and δ for each zone is satisfied.

1. For $\theta_i < \mu^* - 2\varepsilon$,

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta = \frac{1}{12\lambda\varepsilon}\left[(\lambda Q + \beta + \varepsilon)^3 - (\lambda Q + \beta - \varepsilon)^3\right] = \frac{(\lambda Q + \beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda} \geq \delta$$

   and the first inequality is satisfied.

2. The proof for this region is identical to the one for the case $\beta = 0$ and is hence omitted.

3. We shall verify the inequalities for the next three regions together.

   Using the same arguments as in Step III of Proposition 2, we note that the slopes of the two functions, $\pi(\theta_i, \mu^*)$ and $\pi(\mu^*)$ must converge as $\theta_i \longrightarrow \mu^*$ and in particular $\pi(\theta_i, \mu^*)$ must be declining at $\theta_i = \mu^*$. These statements taken together imply that $\pi(\theta_i, \mu^*)$ must have at least one stationary point that is a maximum in the interval, $[\hat{\theta}(\mu^*) - \beta - \varepsilon, \mu^*]$ which includes the region $[\hat{\theta}(\mu^*) - \beta - \varepsilon, \mu^* - \beta]$.

We therefore check the roots of the derivative of $\pi(\theta_i, \mu^*)$ with respect to θ_i.

The derivatives of the first and the second term of $\pi(\theta_i, \mu^*)$ with respect to θ_i are the same as the derivatives of the first and second terms of $\pi(\theta_i, \theta^*)$ in Proposition 2.

The derivative of the third term is given by

$$\frac{1}{4\lambda\varepsilon}\left[3\left(p-\theta_i\right)^2 - 2\left(p-\theta_i\right)\left(\sqrt{\left(p-\theta^*\right)^2 + 4\varepsilon\lambda Q} + \beta\right)\right]$$

which is more conveniently written as,

$$\frac{1}{4\lambda\varepsilon}\left[3\left(p-\theta_i\right)^2 - 2\left(p-\theta_i\right)\left(p + \varepsilon - \hat{\theta}(\mu^*) + \beta\right)\right]$$

As before, these terms can be combined to get

$$\frac{d\pi(\theta_i, \mu^*)}{d\theta_i} = \frac{1}{2\lambda\varepsilon}\left[\theta_i^2 - 2\left(p - \frac{\lambda Q + \varepsilon + \beta}{2}\right)\theta_i + \Omega\left[\mu^*, \beta\right]\right] \tag{40}$$

where

$$\begin{aligned}\Omega\left[\mu^*, \beta\right] \equiv \left(\mu^* - \varepsilon\right)^2 &- \frac{\left(\hat{\theta}\left(\mu^*\right) + \mu^* - \varepsilon\right)^2}{4} + 2\varepsilon\lambda Q \log\left[1 - \frac{\hat{\theta}\left(\mu^*\right) - \left(\mu^* - \varepsilon\right)}{2\varepsilon}\right] \\ &+ \frac{p^2 - \left(\lambda Q + \mu^* - \varepsilon\right)^2}{2} + p\left(\hat{\theta}\left(\mu^*\right) - \varepsilon - \beta\right)\end{aligned}$$

Derivative (40) has two roots given by

$$\theta_{1,2}^R\left(\mu^*\right) \equiv \left(p - \frac{\lambda Q + \varepsilon + \beta}{2}\right) \pm \sqrt{\left(p - \frac{\lambda Q + \varepsilon + \beta}{2}\right)^2 - \Omega\left[\mu^*, \beta\right]},$$

Using the same steps as in the case of $\beta = 0$ in Proposition 2, we show that both roots cannot be less than $\mu^* - \beta$ because of a contradiction.

We next show that $\frac{d\pi(\theta_i, \mu^*)}{d\theta_i} < 0$ for $\theta_i \in [\mu^* - \beta, \hat{\theta}(\mu^*) - \beta + \varepsilon]$. If the last statement is true, then the necessary maxima of $\pi(\theta_i, \mu^*)$ lies in the interval, $[\hat{\theta}(\mu^*) - \beta - \varepsilon, \mu^* - \beta]$ and is unique.

As the form of the function, $\pi(\theta_i, \mu^*)$, is identical over the sub-intervals $[\mu^* - \beta, \mu^*]$ and $[\mu^*, \hat{\theta}(\mu^*) - \beta + \varepsilon]$, the arguments put forth for the function, $\pi(\theta_i, \theta^*)$ for region 4 in Proposition 2 apply and $\frac{d\pi(\theta_i, \mu^*)}{d\theta_i} < 0$ for the entire interval $\theta_i \in [\mu^* - \beta, \hat{\theta}(\mu^*) - \beta + \varepsilon]$.

Thus $\pi(\theta_i, \mu^*)$ has a unique maxima in $[\hat{\theta}(\mu^*) - \beta - \varepsilon, \mu^* - \beta]$. Hence all the three required inequalities for regions 3, 4 and 5 are satisfied.

6. The required inequality follows from the same argument provided for region 5 in Proposition 2.

12.1.1 Proof of Proposition 5 part 2

We need to show that the solution $\mu^* = p - \sqrt{2\lambda\delta}$ is an equilibrium.

As we are looking for an equilibrium under which $r(\theta, \mu^*) = 0$ for all $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$, at the value of θ at which the equilibrium permit price falls to zero, all firms are active. Hence the value of θ at which the equilibrium permit price equals zero is given by $\hat{\theta} = p - \lambda Q < \mu^* + \beta - \varepsilon$.

The individual profit function takes on three different forms depending on three zones in which θ_i may lie. We discuss these forms and the condition that each form must satisfy for μ^* to be equilibrium. We also simultaneously show that these conditions are met.

1. For $\theta_i < \mu^* - \beta - 2\varepsilon$, we require

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta > \delta$$

   The condition has been shown to be satisfied in Step III of the previous Proposition 5.

2. For $\mu^* - \beta - 2\varepsilon \leq \theta_i \leq \mu^* - \beta$, the profit function and the required condition are

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{p-\lambda Q} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\varepsilon} \int_{p-\lambda Q}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2 \, d\theta > \delta$$

   Upon simplification,

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \left[\frac{(p - \theta_i)^3}{3} - \frac{(\lambda Q + \beta - \varepsilon)^3}{3} + (p - \theta_i)^2 (\theta_i + \beta + \varepsilon - p + \lambda Q) \right]$$

   The expression equals $\frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}$ when $\theta_i + \beta + \varepsilon = p - \lambda Q$, implying that the profit function is continuous at this value of θ_i.

   It is easy to show that the derivative of the function with respect to θ_i is negative. Hence, $\pi(\theta_i, \mu^*)$ is strictly declining in θ_i through the range under consideration. Since $\pi(\theta_i, \mu^*) \longrightarrow \delta$ as $\theta_i \longrightarrow \mu^*$, the condition is satisfied.

3. When$\mu^* - \beta < \theta_i < \mu^*$, we must have

$$\pi(\theta_i, \mu^*) = \frac{1}{2\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} \frac{1}{2\lambda} (p - \theta_i)^2 \geq \delta$$

   Since $\frac{1}{2\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} \frac{1}{2\lambda} (p - \theta_i)^2 = \frac{1}{2\lambda} (p - \theta_i)^2$, and the latter function is strictly declining in θ_i, the condition is satisfied, because at $\theta_i = \mu^*$, $\frac{1}{2\lambda} (p - \theta_i)^2 = \delta$.

4. When $\mu^* < \theta_i$, we must have

$$\pi(\theta_i, \mu^*) = \frac{1}{2\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} \frac{1}{2\lambda} (p - \theta_i)^2 < \delta$$

By the same arguments as in the previous step, the condition is satisfied.