Exurban Development

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Abstract
Exurbia, the rural area beyond the built-up urban and contiguous suburban area, is being developed rapidly with attendant losses in habitat and ecosystem services. This paper analyzes a spatial dynamic model with two production technologies for residential development—municipal sewer service for suburban development and septic systems for exurban development. In outlying agricultural areas, the additional sewer extension costs can significantly reduce the value of agricultural land in suburban use. Exurban development, while at lower density, can occur immediately and requires only the onsite conversion costs of septic systems. Hence, the willingness to pay for exurban use from households with higher preferences for lot size may exceed the agricultural landowner's reservation price on future suburban use for a range of distances from the city boundary. This results in a "feasible zone" for exurban leapfrog development and another fundamental reason for scattered development in the urban-rural fringe.

Keywords: Land use; sprawl; urban-rural fringe; leapfrog development; dynamic spatial modeling

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1. Introduction

The zone of exurban development is much larger than the combined footprint of urban and suburban development [12,22]. Although the majority of people in the United States live in the built-up urban and suburban areas, these land uses only occupied 1.9% of the land area in 1992 [2].[1] Sutton et al. [22] used nighttime satellite imagery and found that exurban development occupies 14% of the land area. Exurban large-lot development poses a greater threat to farmland loss than urban and suburban development [12].[2] Ecological studies have demonstrated that native species have reduced survival in the vicinity of exurban homes whereas nonnative species increased [9,11,17]. Exurban development was also found to have a larger overall impact on sediment levels in spawning streams for endangered salmonid species than did urban development because it was more likely to leapfrog into watersheds with intact habitat [16].

Heimlich and Anderson [12] argue that there are "two kinds of growth". One reason is that urban and suburban development typically require municipal sewer and water lines to be extended before higher density development (>1 housing unit per acre) is allowed. Meanwhile, exurban development (≤ 1 housing unit per acre) is invariably serviced by privately provided septic systems and groundwater wells that, due to public-health concerns, mandate adequate spacing between them.[3] Using a parcel-level model in Sonoma County, California, Newburn and Berck [19] demonstrate that suburban development is largely constrained within sewer and water service areas (SWSAs), whereas exurban development leapfrogs well beyond the SWSAs. Because septic systems readily allow homes to be noncontiguous, exurban development is more likely to cause fragmentation and loss of intact agricultural and forested areas.

We analyze a theoretical spatial dynamic model in which two production technologies exist for residential development—municipal sewer service for suburban use and septic systems for exurban use. Suburban use in outlying agricultural areas imposes an additional cost of contiguity because sewer infrastructure must be extended from the existing service boundary. These additional sewer extension costs, or the waiting time until population growth causes the city boundary to expand into an outlying area, can significantly reduce the value of agricultural land in suburban use. Exurban use, while at lower density, can occur immediately and requires only the onsite conversion costs. Our paper is the first, to our knowledge, to incorporate these two sewage production technologies into a theoretical model of land-use development.

Three main results are found. First, suburban development that prematurely extends sewer service beyond the boundary is more costly than optimal suburban development contiguous to the city boundary. Second, the land value in suburban use declines rapidly from the city boundary into agricultural areas, especially when the city radius is small. The willingness to pay (WTP) for exurban use from households with higher preferences for lot size may exceed the agricultural landowner's reservation price for a range of distances from the existing city boundary. This range defines the "feasible zone" for exurban leapfrog development. Third, a household with a higher WTP for lot size can afford to live in exurbia but only during the early phases of city development. While the city radius is small, the additional land and commute costs required to live on an exurban lot, compared to a suburban lot at the city boundary, represent a relatively small portion of household income. But these additional costs increase significantly through time as the city becomes larger. Therefore, there will eventually come a time when exurban use is no longer feasible at any location.

Four fundamental forces underlie models used to describe residential land-use patterns in the urban fringe: (1) commuting costs with varying capital intensity; (2) household sorting into jurisdictions based on public goods and taxes; (3) land-use externalities; and (4) sewage production technologies.[4] Early explanations of leapfrog development in urban economics included models that combine varying density or land use with assumptions of both durable capital and perfect foresight [1,18,20,25,28]. Ohls and Pines [20] used these assumptions to demonstrate why land inside the urban area may be withheld from early development as a result of intertemporal decision making in a growing city. Mills [18] later extends the idea in [20] by incorporating uncertainty and heterogeneity into the expectations of developers. Wheaton [28] allows for changing conditions in income, commute costs, and population and provides scenarios where development occurs from the edge of the city inward. This sequencing pattern shows that leapfrog development can be a pareto efficient outcome.

Tiebout's [24] local public finance model provides the conceptual basis for why heterogeneous households sort into jurisdictions based on their demand for public goods and taxes. Although "flight from blight" is one of the primary forces for urban decentralization, Tiebout abstracts from any explicit representation of space. Building on the locational equilibrium model developed by Epple and Sieg [7], Walsh [27] analyzes the endogenous formation of open space amenities within different jurisdictions. His results indicate that open space provides positive welfare effects in more developed areas, but negative effects in rural areas. Wu [30] integrates Tiebout-style household sorting and exogenous open space (e.g., parks) into an Alonso-Mills-Muth model. He essentially unifies forces (1)-(3) mentioned above. This theoretical model illustrates how protected open space can generate leapfrog development patterns when travel costs fall below a certain level or incomes rise above a certain level.

According to spatial hedonic studies, land-use externalities from surrounding open space also operate at highly local scales such as hundreds of meters [10,13], not only at the scale of jurisdictions [23,27] or major protected areas [29,30]. Similarly, Irwin and Bockstael [14] found that negative spatial spillovers from development can reduce the likelihood of development on neighboring undeveloped parcels, resulting in more fragmented development patterns. Given this empirical evidence, Turner [26] formulates a game theoretic model of micro-scale landowner interaction, where residents receive higher utility if they neighbor a vacant parcel. Local land-use externalities complicate theoretical models since an agent must consider the location of all other agents and land prices when determining their location decisions. Despite the need for highly stylized assumptions, these models provide interesting patterns of leapfrog development.

In sum, recent theoretical models have attempted to unify two or more of the forces (1)-(3) listed above. None of these models have incorporated sewage production technologies (force 4), which is the purpose of our theoretical model developed here.[5] Our model is closely related to Capozza and Helsley [3].[6] However, we consider two residential alternatives and two types of households that differ only in their WTP for lot size. In this manner, our model resembles earlier models [1,18,20], except that we explain why sewage production technologies are essential to understand land-use development in the urban fringe. Hence, this is the first attempt to integrate sewage production technologies and urban economic models for commute costs with durable capital and varying density (forces 1&4). We admittedly do not include local land-use externalities and household sorting over jurisdictions (forces 2&3). We leave this to future work to provide a unified theory that integrates all four forces.

2. The model

This model highlights the role of sewer and septic production technologies on exurban leapfrog development. We assume that the land area is a homogeneous plain with a monocentric city, and locations are indexed by their distance x from the central business district (CBD). Households commute to work at the CBD incurring a constant cost per unit distance *k*. There are two residential lot sizes, namely suburban use with smaller lot size q^s and exurban use with larger lot size q^e. The cost structure for suburban use includes onsite improvement costs and is also a function of distance to the existing city boundary because suburban use is dependent on municipal sewer service. Meanwhile, the cost structure for exurban use has onsite improvement costs but is independent of distance *x*.

There are two types of households that are identical except that a high-type household *h* has a larger willingness to pay (WTP) for lot size than a low-type household *l*. All households are assumed to have a Cobb-Douglas instantaneous utility function from the consumption of a composite numeraire good *z* and lot size, where high-type households have a higher share parameter on lot size. Households all have periodic income $y(t)$ with present value *Y* and make optimal decisions on location and lot size to maximize their present value of discounted utility. Suburban and exurban conversion decisions are irreversible, and capital is perfectly durable with no deterioration. Landowners have perfect foresight, and the agricultural landowner's conversion decision is made to maximize the present value of land.

In the remainder of the model section, we first outline the baseline model for the low-type household *l* in Section 2.1. The population of low-type households is assumed to grow exogenously and occupies the suburban lot size. In Sections 2.2-2.4, we provide the model that includes the market equilibrium and land allocation when introducing both household types (high and low WTP for lot size) and both lot sizes (exurban and suburban).

2.1. Value of agricultural land in suburban use

The baseline model for the population of low-type households is similar to Capozza and Helsley [3] (hereafter, CH model). In contrast to their emphasis inside the city area, we focus on the value of agricultural land outside that area. Additionally, we account for sewer main extension. The $N^{l}(t)$ identical low- type households get instantaneous utility u from the consumption of a composite numeraire good z and a fixed suburban lot size q^{s}. Households purchase these goods with consumption expenditure $b(t)$. The present value of $b(t)$ equals the present value of periodic income $y(t)$.[7] Growth of future suburban development requires centralized infrastructure, particularly sewers, to be extended radially. The radial extent of the suburban area at time t is $x^{s}(t)$.

Both onsite and offsite improvement costs are required to convert from agriculture to suburban use. Onsite land improvement costs per area C^{so} include the fixed costs to establish roads, sidewalks, street lighting, and onsite sewer and water connections. Offsite sewer main extension costs at the margin from the city boundary incur a constant cost per area that, for convenience, we include in the onsite improvement cost C^{so}. In addition to this incremental sewer expansion, offsite costs may also include extending sewer main lines to an agricultural parcel located at some distance beyond the current city boundary, $x > x^{s}(t)$. In this region, these fixed costs would be $C^{sm}\left(x - x^{s}(t)\right)$ at location x, where C^{sm} is the cost per unit distance for extending the sewer main lines. If the parcel has land area L, then the number of suburban lots being served is L/q^{s}. On a household basis, the sewer main extension cost per unit distance is $C^{sm}(x)q^{s}/L$. Hence, total improvement costs per area for converting land to suburban use are

$$C^{s}(x,t) = C^{so} + \frac{C^{sm}\left(x - x^{s}(t)\right)}{L}, \qquad x > x^{s}(t) \tag{1}$$

and C^{so} otherwise.

Suburban rent, $R^s(x,t)$, is the payment per area at time *t* for land improved for suburban use at location *x*. The instantaneous budget constraint can be solved for the numeraire good at time *t*: $z^s(x,t) = b(t) - R^s(x,t)q^s - kx$. The numeraire good includes the house; whereas, suburban rent includes land and improvements. Hence, the model focuses on expenditures required for an improved lot, while the amount spent on the housing structure is incidental to the analysis. A household at the city boundary must be equally well off as when residing at any other location within the city area. Let $u^l\left[z(x,t), q^s\right]$ be the instantaneous utility for the low-type household *l* on a suburban lot q^s for location *x* at time *t*, such that $u^l\left[z(x,t), q^s\right] = u^l\left[z\left(x^s(t), t\right), q^s\right]$ for $x \le x^s(t)$. Since q^s and $b(t)$ are the same at all locations, utility is maximized only if $z(x,t) = z\left(x^s(t), t\right)$. Hence, in equilibrium the rents adjust so that the households have the same level of utility at all occupied locations and the suburban rent gradient is

$$R^s(x,t) = R^s\left(x^s(t), t\right) + \frac{k}{q^s}\left(x^s(t) - x\right). \qquad (2)$$

The agricultural landowner's conversion time decision at location *x* is made to maximize the present value of land. The present value at time t_0 of agricultural land converted to suburban use is

$$V^{as}(x,t_0) = \max_t \left\{ \int_{t_0}^{t} R^a e^{-r(m-t_0)} dm + \int_t^{\infty} R^s(x,m) e^{-r(m-t_0)} dm - C^s(x,t) e^{-r(t-t_0)} \right\}, \qquad (3)$$

where *r* is the discount rate and R^a is a constant agricultural rent. Let t^s be the optimal suburban conversion time. The first term in (3) is the present value of agricultural use from t_0 until t^s. The second term is the present value of suburban use from conversion onward. The last term is the cost of suburban conversion discounted to the present.

The optimal conversion time is either t_0 or is given by the first-order condition (FOC) for (3) using Leibniz's rule

$$R^s(x,t) - R^a - rC^s(x,t) = 0, \qquad x \le x^s(t) \tag{4}$$

$$R^s(x,t) - R^a - rC^s(x,t) + \frac{C^{sm}}{L}\frac{dx^s(t)}{dt} = 0, \qquad x > x^s(t). \tag{5}$$

The left-hand side of (4) decreases with distance *x* because the suburban rent in (2) decreases with distance, and the sewer extension costs in (1) increase with distance. Additionally, the left-hand side of (5) decreases with distance *x* for the same reasons and because the last term in (5) is independent of distance *x*. Therefore, it is optimal to develop at a single distance at a given time. If the location were to occur beyond the city boundary, then it would have been optimal to have developed at the boundary earlier. Hence, optimal suburban development occurs contiguous to the current city boundary. Given that suburban development occurs at the boundary, then (4) is the only case that occurs and simplifies to

$$R^s\left(x^s(t),t\right) = R^a + rC^{so}. \tag{6}$$

Hence, the suburban rent at the city boundary is equal to the forgone agricultural rent plus the cost of borrowing capital for onsite land improvements.

Sewer main extension costs create a discontinuity in the suburban rent gradient at the city boundary. Substituting (6) into (2), the suburban rent function within the city area is

$$R^s(x,t) = R^a + rC^{so} + \frac{k}{q^s}\left(x^s(t) - x\right), \qquad x \le x^s(t). \tag{7}$$

In contrast, the suburban rent function for an agricultural parcel beyond the city boundary is

$$R^s(x,t) = R^a + rC^{so} + \left(\frac{k}{q^s} + \frac{rC^{sm}}{L}\right)\left(x^s(t) - x\right), \quad x > x^s(t). \tag{8}$$

The new term, rC^{sm}/L, is the opportunity costs of capital on sewer extension, which reduces the returns in suburban use. Consequently, the rent gradient within and outside the city boundary, respectively, is $-k/q^s$ for $x \le x^s(t)$ and $-k/q^s - rC^{sm}/L$ for $x > x^s(t)$. The suburban rent gradient is steeper outside the boundary because there are both sewer and commute costs.

The population of low-type households $N^l(t)$ grows exogenously (i.e., closed-city model), similar to the CH model.[8] Because the households each consume lot size q^s, the land area needed to accommodate the population is $N^l(t)q^s$. This land area is equal to the integral over distance *x* from zero to the city boundary $x^s(t)$ at time *t*

$$N^l(t)q^s = \int_0^{x^s(t)} 2\pi x \, \eta^l(x)\,dx \qquad (9)$$

where $\eta^l(x)$ is the proportion of radians at distance *x* occupied by this population. Equation (9) shows how to determine the city boundary $x^s(t)$ as a function of the land proportion devoted to low-type households $\eta^l(x)$. Using $x^s(t)$ in the suburban rent function in (7), we can construct the price of undeveloped land $V^{as}(x,t)$ in (3). That is, the optimal behavior of the low-type households and agricultural landowners result in a mapping between land proportion available and land price, just like an inverse demand function.

2.2. Exurban and suburban development

Although it is not optimal to develop land for suburban use well beyond the city boundary, land may be developed for residential use without extending municipal sewer and water infrastructure. This is done with private septic systems and groundwater wells on individual properties. However, public health regulations mandate adequate spacing of septic systems and wells, resulting in a larger lot size for exurban use, $q^e > q^s$. The exurban lot size is the minimum lot size allowed with septic technology, and it is assumed to be fixed.

Additionally, we posit another type of household h that has a higher WTP for lot size, but otherwise is identical to the low-type household l.[9] The population of high-type households $N^h(t)$ grows exogenously (i.e., closed-city model). We assume all households have a Cobb-Douglas instantaneous utility function where $u\left[z(x,t),q\right]=\left(z(x,t)\right)^{1-\alpha}q^{\alpha}$. High-type households have a higher share parameter on lot size than the low-type households, $\alpha^h > \alpha^l$, reflecting their higher WTP for lot size. We assume that the low-type households have such a low WTP for lot size that they choose to live on the smaller suburban lot q^s at or within the city boundary. By driving the expansion of the city boundary, the low-type population determines the price of agricultural land in suburban use, $V^{as}(x,t)$, as outlined in Section 2.1.

The exurban rent, $R^e(t)$, is the payment per area at time t for land improved for exurban use at location x, and the exurban boundary at time t is $x^e(t)$. If a high-type household h locates on an exurban lot q^e at the boundary, they must be equally well off as residing at any location x, $u^h\left[z(x,t),q^e\right]=u^h\left[z\left(x^e(t),t\right),q^e\right]$ for $x \le x^e(t)$. Because q^e and $b(t)$ are the same at all exurban locations, utility is the same only if $z(x,t)=z\left(x^e(t),t\right)$. Hence the exurban rent is

$$R^e(x,t)=R^e\left(x^e(t),t\right)+\frac{k}{q^e}\left(x^e(t)-x\right) \tag{10}$$

which is analogous to the suburban rent gradient in (2). The present value at time t_0 of agricultural land converted to exurban use is

$$V^{ae}(x,t_0)=\max_t\left\{\int_{t_0}^{t}R^a e^{-r(m-t_0)}dm+\int_t^{\infty}R^e(x,m)e^{-r(m-t_0)}dm-C^{eo}e^{-r(t-t_0)}\right\} \tag{11}$$

where C^{eo} is the onsite land improvement costs per area for exurban use with a septic system. Taking the FOC of (11) with respect to the exurban conversion time and combining with (10), yields the exurban rent function

$$R^e(x,t) = R^a + rC^{eo} + \frac{k}{q^e}\left(x^e(t) - x\right), \quad x \le x^e(t) \tag{12}$$

The agricultural landowner at location *x* chooses the use that provides the highest present value, $V(x,t) = \max\left\{V^{as}(x,t), V^{ae}(x,t), V^a(x,t)\right\}$, where $V^a(x,t) = R^a / r$ is the present value of remaining exclusively in agricultural use. When agricultural land is converted to both suburban and exurban use at location *x*, then the value of agricultural land is

$$V(x,t) = V^{as}(x,t) = V^{ae}(x,t) \ . \tag{13}$$

Otherwise, some agricultural landowners would receive a higher price for land than other agricultural landowners at the same distance *x*.

The high-type household chooses a location to maximize its time discounted integral of instantaneous utility. Letting the rate of utility discount be ρ, the utility for either household type arriving at time *t* and choosing location *x* is

$$J(x,t,q) = \int_t^{\infty} u\left(z(x,m), q\right) e^{-\rho(m-t)} dm \, . \tag{14}$$

A household maximizes utility $J(x,t,q)$ by choosing either of the two lot size options, its location *x*, and its numeraire consumption time path $z(x,t)$.

The utility is constrained by household income $y(t)$, which has permanent income of $Y = \int_t^{\infty} y(m) e^{-r(m-t)} dm$. The permanent income is spent on the numeraire good, lot purchase, commute costs, and improvement costs. These expenditures at any time *t* are equal to $b(t)$. For the suburban lot q^s at location *x*, the sum of these latter three expenditures respectively is

$$E(x,t,q^s) = q^s V(x,t) + \frac{kx}{r} + q^s C^s(x,t) \, . \tag{15}$$

In Section 2.1, we showed that a low-type household has the same periodic expenditures on suburban land rent plus commuting costs in (7) for any location at or within the city boundary, $x \le x^s(t)$. Therefore, the present value expenditures for suburban use $E(x,t,q^s)$ are also the same for the region $x \le x^s(t)$. Due to the additional cost of sewer extension in (8), then $E(x,t,q^s) > E(x^s(t),t,q^s)$ for $x > x^s(t)$. Hence, $E(x,t,q^s)$ for suburban use is minimal at the boundary $x = x^s(t)$. The three expenditures for an exurban lot q^e at location *x* are

$$E(x,t,q^e) = q^e V(x,t) + \frac{kx}{r} + q^e C^{eo}. \tag{16}$$

Later, we will solve for the optimal location $x^e(t)$ for the choice of the exurban lot q^e.

Using the definition of present value of consumption, the household budget constraint is

$$\int_t^\infty z^i(x,m)e^{-r(m-t)}dm = Y - E^i(x,t,q^i), \tag{17}$$

where *i*= *e*, *s* represents exurban or suburban use, respectively. The present value of numeraire consumption if located at *x* and arriving at time *t* is

$$Z^i(x,t) = Y - E^i(x,t,q^i). \tag{18}$$

The household's problem is to choose the lot size option *i* = *e* or *s*, choose location *x*, and choose numeraire consumption $z(x,t)$ to maximize $J(x,t,q)$ subject to the budget constraint (17).

In order to get a closed-form solution for utility $J(x,t,q)$, we make a further assumption about the time preference of utility, specifically that $r = \rho$. With $r = \rho$, numeraire consumption is constant, $z(x,t) = rZ(x,t)$. Since consumption over time and lot size are constant for a given household, the household's utility $u\left[z(x,t),q\right]$ is also constant. This is a consequence of the permanent income hypothesis, and it is independent of the time path of the household's income, despite the increasing rent. These conclusions flow from the dynamic nature of the problem and the assumption that households can borrow or lend at rate r, which are the assumptions required to perform this analysis on a present value basis.

We now construct the present value of utility for the exurban lot at an arbitrary location x and with the maximized numeraire consumption $z(x,t)$. Using the Cobb-Douglas utility, the numeraire consumption is constant over time, such that $J(x,t,q^e) = Z(x,t)^{1-\alpha}(q^e/r)^{\alpha}$ for exurban use. Since the suburban lot choice q^s has minimum expenditure at the city boundary $x^s(t)$, the solution to the consumption problem subject to the intertemporal budget constraint, gives the maximized present value utility $J(x^s(t),t,q^s) = Z(x^s(t),t)^{1-\alpha}(q^s/r)^{\alpha}$ for suburban use.

Before determining the optimal exurban location $x^e(t)$, we find the feasible set of exurban locations at time t where the exurban lot is preferable to the suburban lot at the city boundary. We compare these two choices using the equivalent variation $W(x,t)$, which is the amount of money the household would give up with the exurban lot at location x to obtain the same utility as residing on the suburban lot at the boundary. Using this equal utility condition, equivalent variation is defined as $\left(Z(x,t) - W(x,t)\right)^{1-\alpha}(q^e/r)^{\alpha} = \left(Z(x^s(t),t)\right)^{1-\alpha}(q^s/r)^{\alpha}$.

Using (15), (16), and (18), we solve for the equivalent variation

$$W(x,t) = (1-Q)Y - \left(q^e V(x,t) + kx/r + q^e C^{eo}\right) + Q\left(q^s V(x^s(t),t) + kx^s(t)/r + q^s C^{so}\right), \quad (19)$$

where $Q = \left(\dfrac{q^s}{q^e}\right)^{\alpha/1-\alpha}$. The feasible, but not necessarily optimal, set of locations for exurban use is defined by those locations x where $W(x,t) > 0$. The simulations in Section 3.2 show that there can exist a large range of feasible locations for exurban use, particularly when the city size is still small. Yet only one of the locations in the feasible set is optimal for exurban use at time t.

We now determine the optimal exurban location $x^e(t)$. The location *x* that maximizes utility $J(x,t,q^e)$ must maximize present value of numeraire consumption $Z(x,t)$ because utility is increasing in consumption and exurban lot size q^e is fixed. Since permanent income *Y* is constant, this is the same as minimizing the expenditure for exurban use $E(x,t,q^e)$ in (16). The optimal exurban location is derived from the FOC of (16) with respect to distance *x*

$$q^e \frac{\partial V(x,t)}{\partial x} + \frac{k}{r} = 0 . \tag{20}$$

The first term is the marginal benefit of lower land value if the household locates one unit distance farther from the city boundary. The second term is the marginal cost of commuting if the household locates one unit distance farther from the city boundary. The household considers these countervailing effects when making its location decision for exurban use.

We can further refine the FOC on the optimal exurban location after using (13) and then substituting $V^{as}(x,t)$ from (3) into (20) [10]

$$q^e \left[-e^{-r\left(t^s(x)-t_0\right)} \frac{k}{q^s r} \right] + \frac{k}{r} = 0. \tag{21}$$

Hence, the cost of agricultural land decreases with distance according to the value of suburban commute costs, but is discounted by the time until city expansion, $t^s(x)-t_0$. Note that $t^s(x)$ represents the time when the city boundary reaches the present location of optimal exurban use $x^e(t_0)$, whereas t_0 is by definition the time the city boundary is located at $x^s(t_0)$. The optimal exurban location from the FOC in (21) is located such that

$$t^s(x) = t_0 + (1/r)\ln\left(q^e / q^s\right). \tag{22}$$

Hence, the optimal exurban location is $x^e\left(t_0\right)=x^s\left(t_0+\left(1/r\right)\ln\left(q^e/q^s\right)\right)$, meaning that it occurs where the city boundary will be located in $\left(1/r\right)\ln\left(q^e/q^s\right)$ units of time from the present. Therefore, $x^e\left(t_0\right)-x^s\left(t_0\right)>0$ because $q^e/q^s>1$. Fundamentally, this result indicates that optimal exurban development always leapfrogs beyond the current city boundary.

2.3. Household expenditures and sorting through time

In this section, we discuss how the expenditures needed to afford an exurban and suburban lot size change over time as the city boundary expands. We make two points. First, the present value of expenditure at the optimal location for exurban development in (16) is larger than those for suburban development in (15) evaluated at the city boundary, $E\left(x^e\left(t\right),t,q^e\right)>E\left(x^s\left(t\right),t,q^s\right)$, for all time *t*. Second, as the city boundary expands, there exists some time *T* when even a high-type household eventually will not be able to afford the additional expenditures associated with an exurban lot for any location *x* and, thus, sorts into a suburban lot at the city boundary.[11]

We provide only a brief outline of the calculations for this first point.[12] First, we consider the present value of suburban expenditures in (15) evaluated at the city boundary at time t_0, and we use (13) for the land price to substitute $V^{as}(x,t_0)$ from (3) into (15). Then, using further substitution of the suburban rent in (7) into $V^{as}(x,t_0)$, the present value expenditures simplify to

$$E\left(x^s\left(t_0\right),t_0,q^s\right)=\frac{q^sR^a}{r}+\int_{t_0}^{\infty}k\left[x^s\left(m\right)-x^s\left(t_0\right)\right]e^{-r(m-t_0)}dm+\frac{kx^s\left(t_0\right)}{r}+q^sC^{so}. \quad (23)$$

The first term in (23) represents the opportunity costs of forgone agricultural rents for a suburban lot. The second term is the growth in location value for a household at $x^s\left(t_0\right)$, which results from the city boundary expanding beyond this point in the future. The third term is the commute costs from locating at a distance $x^s(t_0)$ from the CBD. The last term is the improvement costs for a suburban lot at the city boundary.

Similarly, we consider the present value of expenditures for an exurban lot in (16) at the present time t_0 for the optimal location $x^e(t_0)$

$$E\left(x^e(t_0), t_0, q^e\right) = \frac{q^e R^a}{r} + \int_{t^s(x)}^{\infty} k\left[x^s(m) - x^e(t_0)\right] e^{-r(m-t^s)} dm + \frac{kx^e(t_0)}{r} + q^e C^{eo} \qquad (24)$$

where $t^s(x) = t^s\left(x^e(t_0)\right)$ is the time when the city boundary reaches $x^e(t_0)$. We now show that $E\left(x^e(t_0), t_0, q^e\right)$ is greater than $E\left(x^s(t_0), t_0, q^s\right)$, discussing the four terms in (24) and (23), respectively. The first term on opportunity costs of agricultural rents is larger in (24) because $q^e > q^s$. The second and third terms can be combined and simplified to yield $\int_{t^s(x)}^{\infty} kx^s(m) e^{-r(m-t^s)} dm$ for (24) and $\int_{t_0}^{\infty} kx^s(m) e^{-r(m-t_0)} dm$ for (23). The former integral is larger than the latter, as can be shown via a change in the variable of integration. The last term for onsite improvement costs are not known generally. According to Frank [8], onsite improvement costs per area are larger for suburban use, $C^{so} > C^{eo}$, but the costs per household are larger for exurban use, $q^s C^{so} < q^e C^{eo}$.

We now turn to the second point and show that, as the city boundary expands, there exists a time t'' such that $J\left(x^s(t''), t'', q^s\right) > J\left(x^e(t''), t'', q^e\right)$, when high-type households will prefer the suburban lot at the city boundary to the exurban lot at the optimal exurban location. To begin, note that $E\left(x^s(t_0), t_0, q^s\right) \geq \frac{q^s R^a}{r} + q^s C^{so} + \frac{kx^s(t_0)}{r}$ because the omitted term in (23) is positive. Hence, $E\left(x^s(t), t, q^s\right)$ increases without bound in $x^s(t)$. Let t' be the time when the city boundary is so large that numeraire consumption vanishes, $Y - E\left(x^s\left(t'\right), t', q^s\right) = 0$. Since

$E\left(x^e\left(t\right), t, q^e\right) > E\left(x^s\left(t\right), t, q^s\right)$, then $Y - E\left(x^e\left(t'\right), t', q^e\right)$ must be negative. By continuity, there

must be an earlier time t'' when $Y - E\left(x^e\left(t''\right), t'', q^e\right) = 0$, and thus, $Y - E\left(x^s\left(t''\right), t'', q^s\right) > 0$

and $J\left(x^s\left(t''\right), t'', q^s\right) > J\left(x^e\left(t''\right), t'', q^e\right) = 0$. Given that there is a time when suburban use is

preferred by high-type households, it is important to find the earliest time, *T*, when this occurs. When the high-type household is indifferent between suburban and exurban use,

$J\left(x^s\left(T\right), T, q^s\right) = J\left(x^e\left(T\right), T, q^e\right)$ and $W\left(x^e\left(T\right), T\right) = 0$. In sum, as the boundary expands, even

the new high-type household will eventually choose suburban use at the boundary rather than the additional expenditures needed for exurban use, and let *T* be defined as this exurban shutoff time.

2.4. Land allocation and market equilibrium

Given the exogenous population growth for high-type households $N^h\left(t\right)$, we now find the

proportion of land this population occupies at distance *x*, $\eta^h\left(x\right)$. This relates to the proportion of

land $\eta^l\left(x\right)$ in (9), defining the land occupied by the low-type households $N^l\left(t\right)$. For time $t < T$,

new high-type households choose lot size q^e at location $x^e\left(t\right)$ for each corresponding location in

the city boundary expansion path $x^s\left(t\right)$, $x^e\left(t\right) = x^s\left(t + \left(1/r\right)\ln\left(q^e/q^s\right)\right)$. The land area needed

to accommodate high-type households is $N^h\left(t\right)q^e$, which is the integral over distance *x* from

zero to the optimal exurban location $x^e\left(t\right)$ at time *t*

$$N^h\left(t\right)q^e = \int_0^{x^e(t)} 2\pi x\,\eta^{he}\left(x\right)dx \quad \text{for } t < T. \tag{25}$$

where $\eta^{he}(x)$ is the proportion of radians that high-type households occupy in exurban use at distance *x*. Since $x^e(t)$ and $N^h(t)$ are known, then (25) implicitly defines $\eta^{he}(x)$. For time $t \geq T$ after exurban shutoff, the optimal decision for the new high-type households is to choose the suburban lot q^s at the city boundary $x^s(t)$. After time *T*, the additional land area needed to accommodate high-type households is $\left(N^h(t) - N^h(T)\right)q^s$. This land area is equal to the integral over distance *x* starting from the city boundary $x^s(T)$ at exurban shutoff time *T* to the city boundary $x^s(t)$ for all time *t* thereafter

$$\left(N^h(t) - N^h(T)\right)q^s = \int_{x^s(T)}^{x^s(t)} 2\pi x\,\eta^{hs}(x)\,dx \quad \text{for } t \geq T \tag{26}$$

where $\eta^{hs}(x)$ is the proportion of radians that high-type households occupy in suburban use at distance *x*. The logic for the high-type household is that of demand: they take $V(x,t)$ as the given price of land and make their best response in terms of proportion of land to be occupied at each distance *x*, such that $\eta^h(x) = \eta^{he}(x) + \eta^{hs}(x)$. The remaining proportion of land at distance *x* will be occupied by the low-type households, $\eta^l(x) = 1 - \eta^h(x)$. In summary, we have shown how to construct a set of prices $V(x,t)$ as a function of the land devoted to low-type households. From $V(x,t)$, we find the proportion of land demanded by high-type households, $\eta^h(x)$. The system is in equilibrium when the proportions of land to be occupied by both household types satisfies $\eta^l(x) + \eta^h(x) = 1$.

3. Simulations on suburban and exurban location decisions and household expenditures

The theoretical model is used to simulate the location decisions for suburban and exurban development. The simulations in Section 3.1 are used to show the differences in the land value for suburban development with and without sewer extension costs. This corresponds to the model outlined in Section 2.1 for the population of low-type households with the assumption of exponential growth. The simulations in Section 3.2 discuss the feasible and optimal locations for exurban development. This corresponds to the model extended in Sections 2.2-2.4 to describe equilibrium and land allocation with two lot size types and two household types.

3.1. Land value in suburban use for contiguous development versus with sewer extension

Here we compare suburban use beyond the city boundary (i.e., including sewer extension costs) to the value of agricultural land with optimal suburban use contiguous to the boundary. First, we derive the closed-form solution for land value, rent, and the city boundary for the case of exponential population growth at rate g for the low-type household as described in Section 2.1. The population of low-type households is $N^l(t) = N^l(t_0)e^{g(t-t_0)}$, where $N^l(t_0)$ is the initial population at time t_0. There is only one type of residential use and one type of household. Hence, this population occupies the entire city area, $\eta^l(x) = 1$ for $x \le x^s(t)$. The city boundary is

$$x^s(t) = x^s(t_0)e^{g(t-t_0)/2}, \tag{27}$$

where $x^s(t_0) = \left(\frac{N^l(t_0)q^s}{\pi}\right)^{1/2}$. The city growth path in (27) may be inverted to find the optimal conversion time

$$t^s(x) = t_0 + (2/g)\ln\left(x/x^s(t_0)\right), \quad x \ge x^s(t_0). \tag{28}$$

After substituting the city growth path from (27) into the suburban rent in (7), we can determine the maximum value of agricultural land in suburban use in (3) [13]

$$V^{as}\left(x,t_0\right)=\frac{R^a}{r}+\left(\frac{x}{x^s\left(t_0\right)}\right)^{-2r/g}\frac{kgx}{2q^s r\left(r-g/2\right)},\qquad x\geq x^s\left(t_0\right). \tag{29}$$

The value of land has two components—present value of exclusively agriculture and the value of development rights in suburban use, labeled the "growth premium" in the CH model. The growth premium is largest at the boundary but declines with distance *x*. This decline is rapid particularly when either the city radius is small or the ratio between the discount rate to city radial growth rate is large.[14] Note that the factor $\left(x/x^s\left(t_0\right)\right)^{-2r/g}$ is equal to the discount factor, $e^{-r\left(t^s-t_0\right)}$, where the suburban conversion time in (28) is expressed in terms of distance *x*. This means the growth premium depends the time until the boundary expands and municipal services reach this area.

We now determine the land value for suburban development with sewer extension beyond the city boundary and compare it to the maximum value for contiguous suburban development. A developer incurs additional sewer extension costs, according to (1), to convert a distant agricultural parcel located at $x>x^s\left(t_0\right)$. Additionally, households at this distant location *x* pay higher commute costs than households located at the boundary. Taking into account these additional costs, the value of suburban development with sewer extension is

$$VN^{as}\left(x,t_0\right)=\frac{R^a}{r}+\frac{kgx^s\left(t_0\right)}{2q^s r\left(r-g/2\right)}-\left(\frac{k}{q^s r}\right)\left(x-x^s\left(t_0\right)\right)-\frac{C^{sm}\left(x-x^s\left(t_0\right)\right)}{L},\ \ x\geq x^s\left(t_0\right). \tag{30}$$

At the boundary, the value of suburban use in (30) is equal to the maximum value in suburban use in (29). But it declines sharply away from the boundary due to sewer and commuting costs.

The simulations below performed in Matlab demonstrate the magnitude of the effect of sewer and commuting costs according to the following simulation parameters. Household income is $50,000 per year and annual commute costs are $500 per roundtrip mile. Sewer main extension costs are $750,000 per mile, taken from [21], which is about $150 per linear foot.

Onsite land improvement costs are the service costs on streets and utilities in Table 8 in [8], which are \$122,600 per acre for suburban use at 5 units per acre (in 2006 dollars). The suburban lot size is 0.2 acres. Agricultural rent is \$1000 per acre, and the discount rate is 5%. The simulation assumes an initial city radius of 0.1 miles. The population has an annual growth rate of 4% and has an upper bound at 20 million households. The simulation is performed over a 1500 year period to approximate present value of land for a long time horizon.

Consider a snapshot of the simulation when the city boundary reaches 3 miles from the CBD, which corresponds to a population of approximately 100,000 households. Figure 1 shows the maximum value in suburban use, $V^{as}\left(x,t_0\right)$ from (29), when suburban development occurs contiguously at the city boundary. We profile how this value declines from the existing boundary into outlying agricultural areas. Figure 1 also includes the land value for suburban development with sewer extension beyond the city boundary, $VN^{as}\left(x,t_0\right)$ from (30), which is calculated for three different sizes on the distant agricultural parcel: 1, 10, and 100 acres.

The maximum value in suburban use is \$120,000 per acre at the city boundary in Fig. 1. This value declines to the value in agriculture exclusively as one travels farther from the boundary. The land value for suburban use with sewer extension in (30) is also \$120,000 at the boundary in Fig. 1. But it declines more steeply than the maximum value. Thus, suburban use with sewer service extension is suboptimal at all locations. Consider an agriculture parcel of 10 acres located outside the city boundary, for example. As seen by a developer who wants to convert this parcel, the land-value gradient is –\$125,000 per mile due to sewer extension costs and commute costs from premature development. That is, sewer and commute costs decrease the land value per acre by \$75,000 and \$50,000, respectively, for each additional mile from the boundary. Within 1 mile of the boundary, the value for suburban use with sewer extension is below the value for agriculture exclusively. The component of the land-value gradient attributable to sewer extension costs becomes less steep when the agricultural parcel size is large. The sewer main component on the land value gradient in Fig. 1 is –\$750,000, –\$75,000, and –\$7,500 per mile for properties of 1, 10, 100 acres, respectively.

Sewer extension costs are even higher if the developer has to compensate intervening agricultural landowners for an easement precluding future development along sewer lines (to provide access for maintenance equipment) or has to extend sewer lines over a longer distance using existing roads to avoid right-of-way access issues. Topography is another consideration. Pumping stations require additional costs to traverse areas with even moderate slopes.

3.2. Feasible and optimal locations for exurban and suburban development

We now calculate the household expenditures required for exurban and suburban development for different city sizes. In these scenarios, smaller (larger) city sizes represent earlier (later) phases in the evolution of cities. The purpose is to assess how city size influences the feasible range of locations for exurban development. Additionally, we show how the optimal exurban location that minimizes expenditures is essentially a tradeoff between the cost of agricultural land (lot purchasing costs) and commute costs.

The following simulations have the same parameters used in Fig. 1. Additionally, the total population has 10% high-type households and 90% low-type households. The population growth rate is 4% for both household types. The exurban lot size is one acre due to the density limit with septic systems. Onsite land improvement costs are \$57,500 per acre (in 2006 dollars) for exurban development at one unit per acre [8]. This is lower on a per area basis than onsite suburban conversion costs, $C^{eo} < C^{so}$, because exurban development has less intensive site improvement costs (e.g., narrower and lower quality roads, lack of sidewalks, etc.). However, these costs are higher on a per household basis, $q^e C^{eo} > q^s C^{so}$, since the exurban lots are five times larger. High-type and low-type households have preferences defined by a Cobb-Douglas utility function with alpha parameter on lot size of 0.10 and 0.04, respectively.

The simulation model is performed in discrete time for a 1500 year time horizon at quarterly time increments and at radial distance increments of 0.001 miles. As noted above, the initial city radius is 0.1 miles and the population has an upper bound at 20 million households. The following steps are used to find the equilibrium for exurban and suburban use: 1) an initial vector for the proportion of land to be occupied in suburban use $\eta^l(x)$ is chosen at each distance *x*; 2) the price of agricultural land in suburban use $V^{as}(x,t)$ is calculated as in section 2.1; 3) equations in sections 2.2-2.4 are used to find the proportion of land to be occupied in exurban use $\eta^h(x)$ and hence a new estimate of $\eta^l(x) = 1 - \eta^h(x)$; and 4) repeat steps 2 and 3 until the root mean square difference of $x^s(t)$ between iterations is less than 0.005 miles.

Consider a snapshot of the simulation when the city boundary has expanded to three miles. Figure 2a shows household expenditures required for exurban use, according to (16), for a city with three-mile radius. The optimal exurban location occurs at 5.8 miles (i.e., 2.8 miles from the city boundary), which minimizes total expenditures at $176,000 per household (Fig. 2a). This optimal location is a tradeoff between higher commute costs farther from the CBD versus higher cost of agricultural land close to the city boundary, as shown in (20).

At the city boundary, the cost of agricultural land to purchase an exurban lot is largest, about $125,000 for a one-acre lot. In this vicinity, the agricultural landowner's reservation price is higher because the time until suburban development is short. At much greater distances, the agricultural landowner would have to wait many decades before the city boundary expands and suburban development occurs; therefore, the cost of agricultural land flattens out and eventually approaches the value of agriculture exclusively of $20,000 per acre (Fig. 2a). Meanwhile, expenditures for the present value of commute costs increases linearly as the household locates farther from the CBD, increasing by $10,000 per mile. Onsite improvement costs for exurban use are not dependent on location. Unlike the cost of contiguity that sewer infrastructure imposes on suburban use, exurban use is more footloose and can occur at the present in the countryside, albeit with a larger lot size due to the septic technology.

Figure 2b shows the equivalent variation (EV) for a high-type household, according to (19), for a city with three-mile radius. The feasible range of locations for exurban use is the region where the equivalent variation is positive (Fig. 2b). The optimal exurban location occurs at 5.8 miles, which corresponds to the city boundary reaching this location about 32 years later. Exurban development, while feasible, would not be optimal close to the city boundary. While the city size is still small, the household with a high WTP for lot size could afford to purchase an acre lot at the city boundary. This household, however, is better off by locating away from the boundary. At the optimum, the price of agricultural land decreases by $65,000 per acre, relative to the boundary, because sewer service would not extend into this area for decades. Meanwhile, the household only incurs an additional $28,000 in commute costs (i.e., $10,000 per mile), thereby resulting in a net savings. If the household locates even further away than the optimum, commuting costs continue to increase linearly while the price of agricultural land declines more gradually. Eventually, the equivalent variation approaches zero at approximately 14.2 miles (Fig. 2b). This location defines the upper bound on feasible exurban development at the present.

In sum, high-type households are 10 percent of the total population and optimally would locate on an exurban lot at 5.8 miles when the city boundary is located at 3 miles (Figs. 2a and 2b). Meanwhile, low-type households are 90 percent of the total population and optimally locate on a suburban lot at the city boundary. In other words, the low-type household is not willing to pay the additional costs needed for an exurban lot (i.e., \$176,000 for an exurban lot at 5.8 miles versus \$79,500 for a suburban lot at 3 miles). According to the market equilibrium and land allocation conditions in (9) and (25), this corresponds to high-type households occupying 22.4 percent of the radians in exurban use at 5.8 miles, while leaving the majority of surrounding land in agricultural use for future suburban use. Low-type households occupy 77.6 percent of the radians as suburban lots at the boundary. Note that exurban use occupies more than 10 percent of the developed land area because the exurban lot size is fivefold larger than the suburban lot size. However, at each given city radius, exurban use is optimally located further away.[15]

As the city becomes larger, the cost of agricultural land increases sharply. One reason is that the growth premium at the city boundary increase proportionally to the city radius, similar to the CH model. Figure 3a shows the household expenditures required for exurban use for a city with five-mile radius. This represents a total population of approximately 250,000 households. The land value in suburban use at the city boundary increased from \$125,000 to \$192,000 per acre for a city with radius of three and five miles, respectively (Figs. 2a and 3a). Additionally, the cost of agricultural land in Fig. 3a declines more slowly with distance because a larger city expands into outlying agricultural areas faster than a smaller city, *ceteris paribus*.

Figure 3b shows the equivalent variation for a high-type household for a city with five-mile radius. The high-type household's equivalent variation for exurban use becomes smaller over time as the city evolves. In fact, the equivalent variation is positive only for locations spanning between 6.7 and 14.8 miles, respectively, defining the lower and upper bounds on feasible exurban use (Fig 3b). An “exurban dead zone” exists between the city boundary at 5 miles and lower exurban bound at 6.7 miles. In this zone, agricultural landowners are better off waiting for imminent suburban development at higher density, and thus, would never sell to develop the property into exurban lots. The optimal exurban location is now located even farther away at 9.9 miles from the CBD (i.e., 4.9 miles from the city boundary).

As the city boundary grows, total expenditures required to live on an exurban lot increase through time for two reasons (Table 1). First, commuting costs increase because the optimal exurban location becomes more distant. Second, the cost of agricultural land at the optimal exurban location is higher for a larger city, increasing from $60,200 to $83,100 per acre for a city with radius of three and five miles, respectively (Figs. 2a and 3a).

Household expenditures are higher for exurban use at the optimum than for suburban use at the city boundary. This is true for each of the three cost components (Table 2). When the city radius is small, the difference is relatively small, so the high-type household is willing to pay the additional costs associated with an exurban lot. As the city evolves, the additional costs for exurban use increase significantly; therefore, there will come a time when exurban use is no longer feasible at any location for the high-type household. After this time, the high-type household would optimally sort into a suburban lot at the city boundary.

To investigate this further, we map the trajectories of the city boundary, optimal exurban location, and upper and lower boundaries on feasible exurban use as a function of time. Figure 4 shows the trajectories of these four boundaries starting from when the city radius is located at 1 mile. During early stages of city growth, the land area within the feasible zone for exurban use is much larger than the city area. When the city radius grows to 3 miles, for instance, the city area is only 28 square miles. The feasible exurban zone, which spans a ring between the city boundary at 3 miles and upper exurban boundary at 14.2 miles, has an area of about 605 square miles. Hence, while the municipal sewer service area is still relatively small, the commutershed has many available sites that would be feasible for exurban development.

An exurban dead zone emerges once the city reaches about 4 miles in radius (Fig. 4). As the city size increases, the exurban dead zone continues to grow because the agricultural landowner's reservation price becomes larger at the boundary and extends farther into countryside. Eventually, after about 92 years, the radius of the city boundary reaches more than 6 miles (i.e., 350,000 households) and exurban use is no longer feasible at any distance. Hence, as the city becomes larger, the newly arriving high-type households would ultimately sort into the suburban lot at the city boundary after this exurban shutoff time.

Exurban use, however, may persist longer than in the simulations for several reasons. First, a larger city may not be able to sustain a high population growth rate, such as 4% used above. With a lower population growth rate, the cost of agricultural land declines more rapidly with distance because the time until suburban conversion is longer and thus would delay the exurban shutoff time. Second, the exurban upper bound extends farther if commute costs are lower. Exurban use would have reduced commute costs with telecommuting or when used as a vacation home. Additionally, as the city area expands, the exurban household could potentially commute to a suburban subcenter at the edge of a larger city rather than the CBD, which would lower commute costs. Third, households with annual income higher than $50,000 (median income in the United States used above) can afford exurban development as the city becomes larger. Lastly, the exurban upper bound extends farther with lower agricultural rents. We used an agricultural rent of $1000 to reflect annual returns from high-value intensive agriculture.

However, annual rents may be an order of magnitude less for ranching or forestry. In this case, it would be relatively easy for a household to outbid even larger acreage (5 or 10 acre lots) rather than the maximum density of one acre allowed by septic systems.

4. Conclusion

The difference between sewer versus septic production technologies is one reason for exurban leapfrog development. Suburban development in outlying areas imposes an additional cost of contiguity because sewer lines must be extended from the existing municipal boundary. With septic systems, an agricultural landowner can convert to exurban use at the present without either investing to extend sewer lines or waiting for the municipal service boundary to expand.

One insight from our model is that areas around smaller towns and cities are the most likely places for exurban leapfrog development. A city has a radial extent of only a few miles when the population is on the order of fewer than 100,000 inhabitants (e.g., Gainesville, FL; College Station, TX). For a small city, the sewer service boundary expands slowly, and agricultural landowners located several miles from the boundary may wait decades for suburban development. The agricultural landowner’s reservation price on future suburban use declines rapidly away from the municipal boundary. Households with a higher WTP for lot size can easily afford the additional land and commute costs needed to live on an exurban lot in the vicinity of a small city. This results in a “feasible zone” for exurban development, and this commutershed for exurban use is much larger than the municipal sewer service area for a small city (Fig. 4).

However, exurban development is less affordable near large cities (e.g., Phoenix, Atlanta). As a city becomes larger, the cost of agricultural land increases for two reasons. First, the growth premium at the city boundary increases proportionally to the city radius. Second, the cost of agricultural land declines more slowly because the sewer service boundary for a large city moves faster into the agricultural area. Therefore, the minimum household expenditure required for exurban development increases over time as the city grows.

The differences between suburban and exurban development are important because the vast majority of land lies outside existing municipal sewer service areas. The urban and suburban footprint detected from LANDSAT imagery only occupied 1.9% of the entire United States in 1992 [2]. This implicitly means that existing sewer service areas are scattered over a small portion of the land area nationwide. If one considers the land area within reasonable commuting distance around each town and city, these commutersheds would together occupy a much larger area. Exurban use may not occupy the entire feasible zone because septic systems easily allow this form of residential development to be noncontiguous. Local environmental amenities, such as a lake or hillside view, further contribute to noncontiguous development. But even a small exurban population may cause a high degree of fragmentation in the urban fringe.

In conclusion, this study aims to draw attention to the dynamics of residential development in the urban fringe, particularly in the vicinity of smaller towns and cities. The extension of municipal sewer service is essential for restructuring rural areas into incorporated cities allowing for suburban and urban development. This restructuring process is often relatively slow in nonmetropolitan counties in the vicinity of smaller towns. Our theoretical model provides a first attempt to explain the role of sewage production technologies in the prevalence of large-lot exurban development in the urban fringe. This effort additionally provides a foundation for future models to establish a unified theory that integrates sewage production technologies with other fundamental forces, such as land-use externalities, Tiebout-style household sorting across local jurisdictions, and commuting costs.

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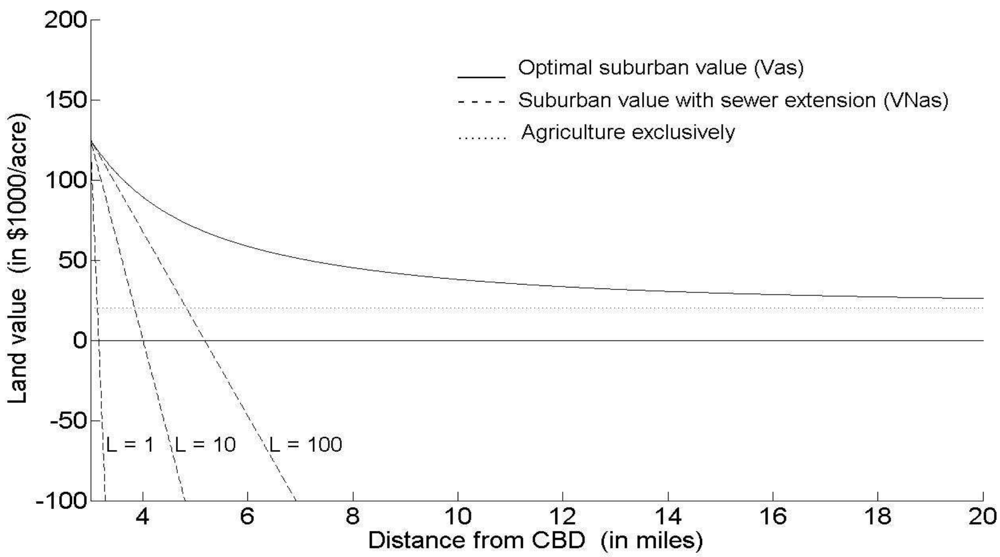

Figure 1. The value of agricultural land for optimal suburban development (Vas) contiguous to the city boundary and for suburban development with sewer extension costs (VNas) to a distant agricultural property with lot size (L) of 1, 10, and 100 acres. The x-axis is the distance from the CBD, starting from the current city boundary at three miles.

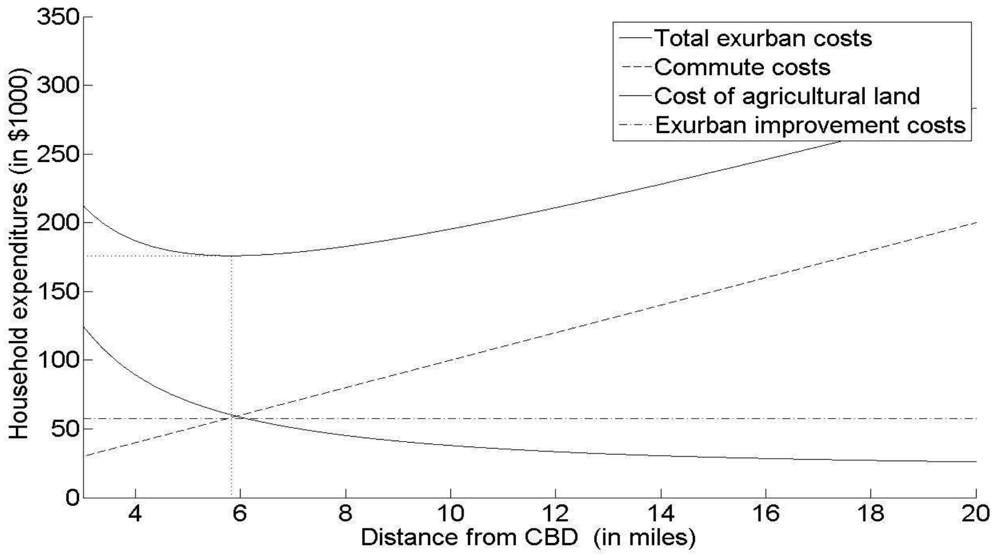

Figure 2a. Household expenditures required for exurban development, including commute costs, lot purchasing costs, and onsite land improvement costs. The x-axis is the distance from the CBD, starting from the current city boundary at three miles.

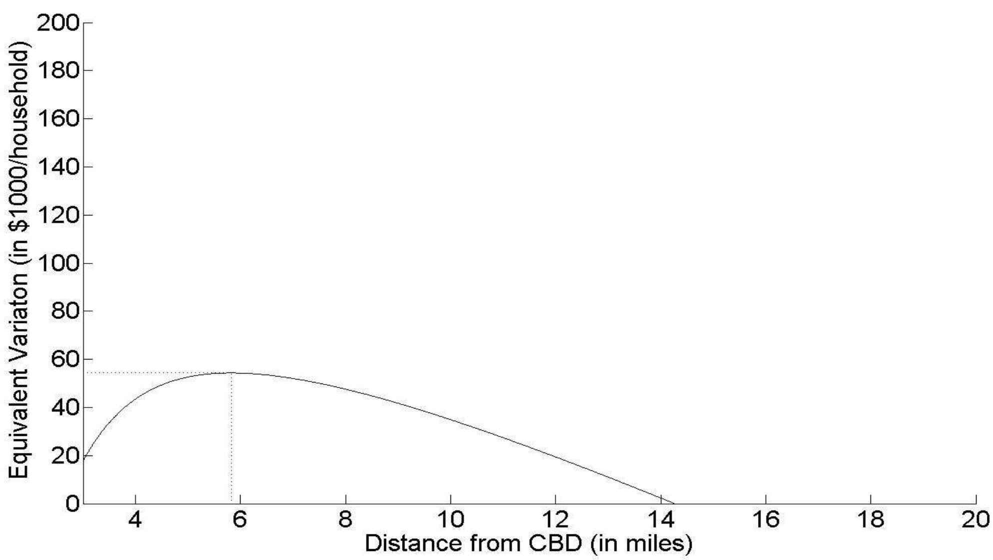

Figure 2b. Equivalent variation for a high-type household showing the feasible range of exurban development. The x-axis is the distance from the CBD, starting from the current city boundary at three miles.

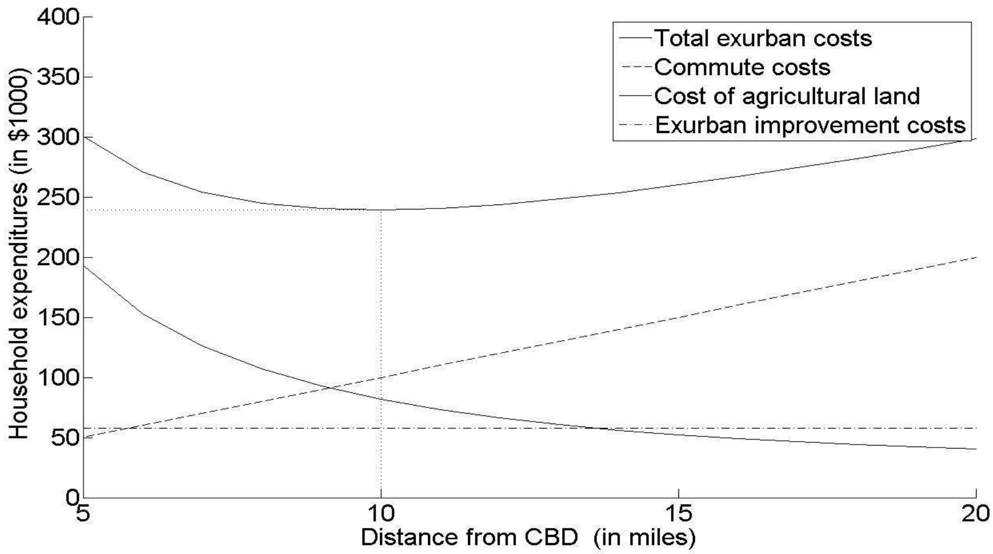

Figure 3a. Household expenditures required for exurban development, including commute costs, lot purchasing costs, and onsite land improvement costs. The x-axis is the distance from the CBD, starting from the current city boundary at five miles.

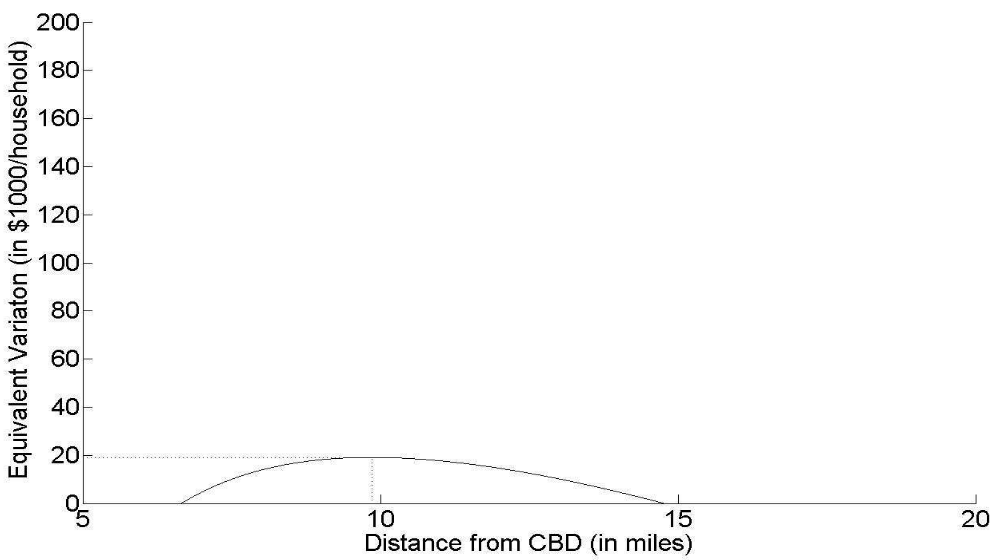

Figure 3b. Equivalent variation for a high-type household showing the feasible range of exurban development. The x-axis is the distance from the CBD, starting from the current city boundary at five miles.

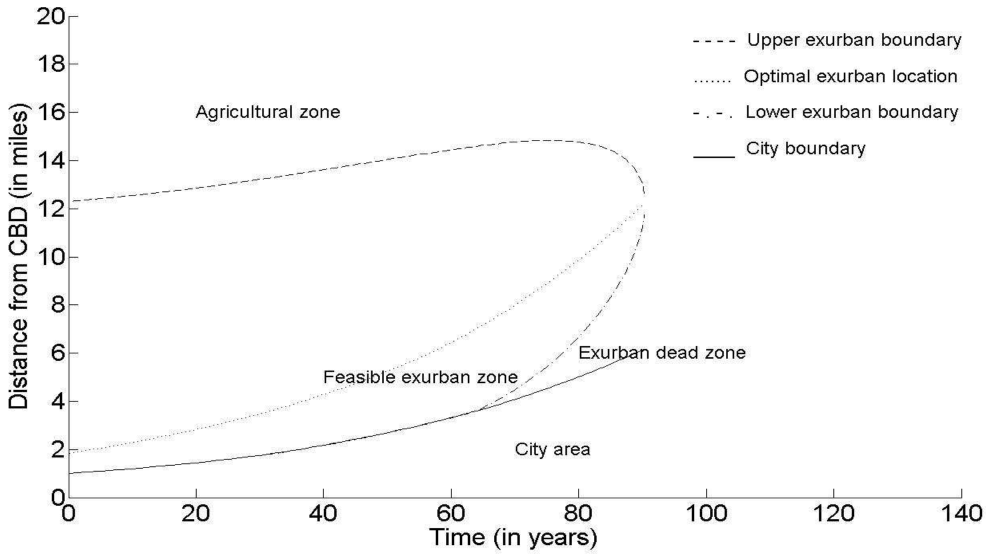

Figure 4. Trajectories for the city boundary, optimal exurban location, and the lower and upper bounds on feasible exurban development.

Table 1: Minimum household expenditures for exurban and suburban lot for different city sizes.

Location (in miles)			**Exurban lot at optimal location (in $1000)**				**Suburban lot at city boundary (in $1000)**			
City boundary	Optimal exurban location	Number of households	Agricultural land costs	Commute costs	Improvement costs	Total costs	Agricultural land costs	Commute costs	Improvement costs	Total costs
1	1.8	10,154	33.4	18.4	57.5	109.3	10.5	10.0	24.5	45.0
2	3.9	42,857	47.2	39.5	57.5	144.2	18.5	20.0	24.5	63.0
3	5.8	91,641	60.2	58.3	57.5	176.0	24.9	30.0	24.5	79.4
4	7.9	160,433	72.1	78.7	57.5	208.3	31.7	40.0	24.5	96.2
5	9.9	246,627	83.1	98.6	57.5	239.2	38.6	50.0	24.5	113.1
6	11.9	353,499	93.7	118.9	57.5	270.1	45.6	60.0	24.5	130.1
7	13.6	472,425	105.3	136.0	57.5	298.8	51.6	70.0	24.5	146.1
8	15.3	606,607	117.0	152.9	57.5	327.4	57.3	80.0	24.5	161.8
9	17.0	755,878	128.5	169.6	57.5	355.6	62.7	90.0	24.5	177.2
10	18.7	923,232	140.1	186.6	57.5	384.2	68.2	100.0	24.5	192.7

Endnotes

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[1] The urban footprint in Burchfield et al. [2] is based on land-cover classifications from LANDSAT imagery, relying on reflected sunlight that can only detect urban and suburban use (> 1 unit per acre). This detection threshold coincidentally is the same as the density restriction for residential use with septic systems. LANDSAT cannot distinguish exurban use (< 1 unit per acre) from extensive land uses (e.g., agriculture, forestland, ranchland).

[2] According to Heimlich and Anderson [12], "nearly 80% of the acreage used for recently constructed housing is land outside urban areas. Almost all of this land (94%) is in lots of 1 acre or larger, with 57% on lots of 10 acres or larger (i.e., 10-22 acres)."

[3] Heimlich and Anderson [12] analyzed residential lot size by sewage disposal for public sewer and septic systems using the American Housing Survey data (see Figure 10 on page 21). "Nearly all lots greater than 1 acre were not sewered."

[4] See Irwin [15] for a review of these four forces in the recent literature.

[5] For instance, Wu and Plantinga [29] develops a model for leapfrog development that integrates forces 1&3 which was later extended in Wu [30] to incorporate force 2 as well. However, both studies implicitly assume that residential density is unconstrained in the region outside the municipal sewer service boundaries.

[6] Our model and Capozza and Helsley [3] both assume that rents are certain. This is a simplification of the model in Capozza and Helsley [4] in which uncertainty in rents is introduced. Capozza and Li [5] later integrate the uncertainty results in [4] when capital intensity is allowed to be endogenously determined.

[7] In section 2.2, we explain how household consumption expenditures $b(t)$ relates to permanent income (or present value of income) Y.

[8] We assume that at some time t^* the population $N^l(t)$ reaches an upper bound $N^l(t^*)$, such that the city boundary reaches an upper bound $x^s(t^*)$ and thus commute costs does not exceed income.

[9] We could have considered different incomes as well as different WTP for lot size. But we will show that, even with the same income, there are conditions under which some households would live in exurban development. Additionally, we consider only the benefits of private open space (yard space); therefore, exurban development would be even more likely if the benefits from landscape amenities were taken into account as in [6,26].

[10] When evaluating the first-order condition $\partial V^{as}(x,t)/\partial x$, we use the envelope theorem for the optimal conversion time, $\partial V^{as}(x,t)/\partial t^s = 0$. The second-order condition is satisfied as well, $-re^{-r(t^s-t_0)}\partial t^s / \partial x < 0$.

[11] In the online supplemental appendix (A1), we examine the model when assuming only a single household type (i.e., homogeneous preferences) and demonstrate that the household sorts into both suburban and exurban use at the same time only for a particular city growth path.

[12] The details on this proof are available in the online supplemental appendix (A2).

[13] We again assume that the population $N^l(t)$ has an upper bound at some time t^* so that the city boundary reaches an upper bound $x^s(t^*)$ and thus commute costs does not exceed income. For this exponential case, the land value is thus only an approximation when the time t^* is large.

[14] For the land value in suburban use to be bounded, $r > g/2$.

[15] In the supplementary appendix, Figure A1 depicts the spatial pattern of exurban and suburban development when the city boundary has expanded to a radius of 3 miles.